AQA Maths Pure Core 3

Mark Scheme Pack

2006-2015

Version 1.0: 0306

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2006 examination - January series

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Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

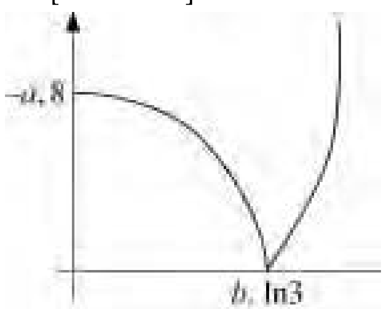
MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\frac{dy}{dx} = 3\sec^2 3x$	M1 A1	 2	for sec $3x$ SC/$3\sec^2 x$ B1
	Alternative Use of product/Quotient rule (M1) $\frac{3\cos^2 3x + 3\sin^2 3x}{\cos^2 3x}$ (A1)			Good attempt Correct
(b)	$\frac{dy}{dx} = \frac{(2x+1)3 - 2(3x+1)}{(2x+1)^2} = \frac{6x+3-6x-2}{(2x+1)^2}$	M1 A1		use of quotient rule
	$= \frac{1}{(2x+1)^2}$	A1	3	AG (no errors)
	Alternative $-2(3x+1)(2x+1)^{-2} + 3(2x+1)^{-1}$ (M1A1) $= \frac{1}{(2x+1)^2}$ (A1)			**Alternative:** $y = \frac{3}{2} - \frac{1}{2}(2x+1)^{-1}$ M1A1 $\frac{dy}{dx} = (2x+1)^{-2}$ A1 $= \frac{1}{(2x+1)^2}$ AG
	Total		**5**	
2	$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,dx$			
	x / y: 1 0.707(1) 1.5 0.478(1) 2 0.333(3) 2.5 0.245(3) 3 0.189(0)	B1 B1		3 correct } **SC** B1 for all correct expressions but all correct } wrongly evaluated
	$A = \frac{1}{3} \times 0.5 \left[\begin{array}{l} y(1) + y(3) + \\ 4(y(1.5) + y(2.5)) + 2(y(2)) \end{array} \right]$	M1		use of Simpson's rule
	$= 0.743$	A1	4	
	Total		**4**	

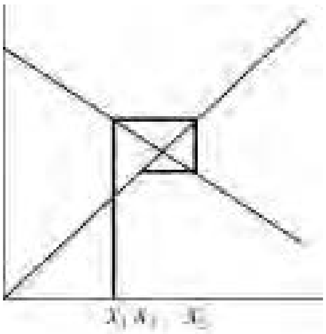
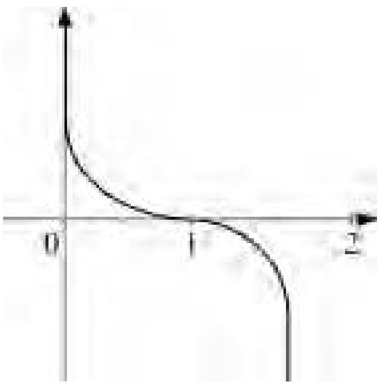
MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\mathrm{f}' = \dfrac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 + 2$	B1	1	
(ii)	$\displaystyle\int \frac{2x^3+1}{x^4+2x}\,\mathrm{d}x$			
	$= \dfrac{1}{2}\ln\left(x^4+2x\right)\ (+\mathrm{c})$	M1 A1	2	For $k\ln\left(x^4+2x\right)$ By substitution $k\ln u$ M1 correct A1
(b)(i)	$u = 2x+1$			
	$\mathrm{d}u = 2\,\mathrm{d}x$	B1		
	$\displaystyle\int x\sqrt{2x+1}\,\mathrm{d}x =$			
	$\displaystyle\int\left(\frac{u-1}{2}\right)\sqrt{u}\,\frac{\mathrm{d}u}{2}$	M1		Must be in terms of u only incl. du
	$= \dfrac{1}{4}\displaystyle\int\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$	A1	3	AG
(ii)	$\displaystyle\int_0^4 \mathrm{d}x = \int_1^9 \mathrm{d}u$	B1		Or changing u's to x's at end
	$\dfrac{1}{4}\displaystyle\int u^{\frac{3}{2}} - u^{\frac{1}{2}} = \frac{1}{4}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]$	M1 A1		
	$= \dfrac{1}{4}\left[\left(\dfrac{2}{5}(9)^{\frac{5}{2}} - \dfrac{2}{3}(9)^{\frac{3}{2}}\right) - \left(\dfrac{2}{5} - \dfrac{2}{3}\right)\right]$			Sight of any of these 3 lines
	$= \dfrac{1}{4}\left[79.2 + 0.2\dot{6}\right]$			
	$= 19.86$			
	$= 19.9$	A1	4	AG
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$2\operatorname{cosec}^2 x = 5(1-\cot x)$			
	$2+2\cot^2 x = 5-5\cot x$	M1		use of $\operatorname{cosec}^2 x = 1+\cot^2 x$
	$2\cot^2 x + 5\cot x - 3 = 0$	A1	2	AG
(b)	$(2\cot x - 1)(\cot x + 3) = 0$	M1		or $2+5t-3t^2=0$ Or in $\tan x$ $(2-t)(1+3t)=0$
	$\cot x = \frac{1}{2}, -3$			
	$\tan x = 2, -\frac{1}{3}$	A1	2	AG
(c)	$x = 1.1,\ -2.0$ } AWRT	B1		Any 2 correct — In degrees: B0
	$x = -0.3,\ 2.8$	B1		Any 3 correct — B1
		B1	3	4 correct — B2
	Total		**7**	
5(a)	$a = -8$	B1		
	$e^{2x} - 9 = 0$	M1		
	$e^{2x} = 9$			
	$2x = \ln 9$			
	$x = \ln 3$	A1	3	AG Condone verification
(b)	$\left(e^{2x}-9\right)^2 = e^{4x} - 18e^{2x} + 81$	B1	1	AG
(c)	$V = \pi\int y^2\,(dx)$	B1		
	$= (\pi)\int e^{4x} - 18e^{2x} + 81\ dx$	M1		
	$= (\pi)\left[\frac{e^{4x}}{4} - 9e^{2x} + 81x\right]_0^{\ln 3}$	M1 A1		1ST or 2nd term correct All correct
	$= (\pi)\left[\left(\frac{e^{\ln 81}}{4} - 9e^{\ln 9} + 81\ln 3\right) - \left(\frac{1}{4} - 9\right)\right]$	m1		Attempt at limits with ln3
	$= \pi[81\ln 3 - 52]$	A1	6	
(d)	 −a	M1 A1F	2	Modulus graph All correct
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{f}(0.5) = -0.875$	M1		
	$\mathrm{f}(1) = 2$			
	Change of sign $\therefore$ root	A1	2	
(b)	$x^3 + 4x - 3 = 0$			
	$4x = 3 - x^3$	B1	1	
	$x = \dfrac{3 - x^3}{4}$			AG
(c)(i)	$x_1 = 0.5$	M1		
	$x_2 = 0.71875$ 0.72 AWRT	A1		
	$x_3 = 0.66$	A1	3	
(ii)		M1		For cobweb, x_1 to curve
		A1		For x_2
		A1	3	All correct
	Total		**9**	
7(a)	$\left(1, \dfrac{\pi}{2}\right)$ OE in decimals	B1		Or for -1 and 1
	$\left(-1, -\dfrac{\pi}{2}\right)$	B1	2	
(b)		M1		Translation in + *ve* x direction
		M1		Correct shape
		A1	3	Correct Graph Through (1,0) touching y – axis
	Total		**5**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	(Range of f) $\geqslant 0$	B1	1	
(b)(i)	$\text{fg}(x) = \dfrac{1}{(x+2)^2}$	B1	1	OE Maybe in part (ii)
(ii)	$\dfrac{1}{(x+2)^2} = 4$			
	$(x+2)^2 = \dfrac{1}{4}$	M1		Or $4(x+2)^2 = 1$
	$x + 2 = (\pm)\dfrac{1}{2}$	M1		$(2x+5)(2x+3) = 0$
	$x = -\dfrac{5}{2}, -\dfrac{3}{2}$	A1 A1	4	
(c)(i)	Not one to one	E1	1	OE
(ii)	$x = \dfrac{1}{y+2}$	M1		$x \Leftrightarrow y$
	$y + 2 = \dfrac{1}{x}$	M1		Attempt to isolate
	$y = \dfrac{1}{x} - 2 \quad \left(\dfrac{1-2x}{x}\right)$	A1	3	
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$y = x^{-2}\ln x$			
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-2}\frac{1}{x} - 2x^{-3}\ln x$	M1 A1 A1		Use of product or quotient each term
	$= \frac{1-2\ln x}{x^3}$	A1	4	Convincing argument $x^{-2} \times \frac{1}{x} = x^{-3}$ AG
(b)	$\int x^{-2}\ln x\,\mathrm{d}x \quad u = \ln x \quad \mathrm{d}v = x^{-2}$	M1		Attempt at integration by parts
	$\mathrm{d}u = \frac{1}{x} \quad v = -x^{-1}$	A1		
	$\int = -\frac{1}{x}\ln x + \int x^{-2}\,\mathrm{d}x$	A1		
	$= -\frac{1}{x}\ln x - \frac{1}{x}(+\mathrm{c})$	A1	4	
(c)(i)	At A, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$			
	$1 - 2\ln x = 0$			
	$\ln x = \frac{1}{2}$	M1		Attempt at $\ln x = k$
	$x = \mathrm{e}^{\frac{1}{2}}$	A1	2	
(ii)	$R = \left[-\frac{1}{x}(\ln x + 1)\right]_1^5$	M1		$R = \Big[\text{ Their}(\mathrm{b})\Big]_1^5$
	$= -\frac{1}{5}(\ln 5 + 1) + (\ln 1 + 1)$	A1		OE
	$= \frac{1}{5}(4 - \ln 5)$	A1	3	convincing argument; AG
	Total		**13**	
	TOTAL		**75**	

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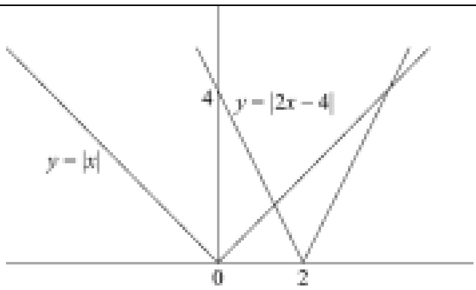
1(a)	$f(2) = -1$ $f(2.1) = +0.161$	M1		both attempted
	change of sign $\therefore 2 < \alpha < 2.1$	A1	2	
(b)	$x^3 - x - 7 = 0$ $x^3 = x + 7$ $x = \sqrt[3]{x+7}$	B1	1	AG
(c)	$x_1 = 2$	M1		
	$x_2 = 2.0801...$	A1		AWRT 2.08
	$x_3 = 2.0862...$			AWRT 2.09
	$x_4 = 2.09$	A1	3	
	Total		**6**	
2(a)	$y = (3x-1)^{10}$ $\frac{dy}{dx} = 10\,(3x-1)^9 \times 3$ $= 30\,(3x-1)^9$	M1 A1	2	M1 for $a(3x-1)^9$ where a = constant
(b)	$\int x(2x+1)^8\,dx$ $u = 2x+1$ $du = 2\,dx$	B1		OE
	$\int = \int\left(\frac{u-1}{2}\right)u^8\left(\frac{du}{2}\right)$	M1		all in terms of u. Condone omission of du
	$= \frac{1}{4}\int u^9 - u^8\,du$			
	$= \frac{1}{4}\left[\frac{u^{10}}{10} - \frac{u^9}{9}\right]$	B1		$p\frac{u^{10}}{10} + q\frac{u^9}{9}$
	$= \frac{(2x+1)^{10}}{40} - \frac{(2x+1)^9}{36}\ (+c)$	A1	4	OE; CAO SC: correct answer, no working/parts in x (B1)
	Total		**6**	

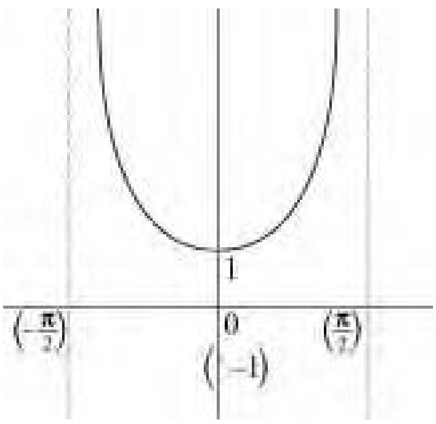
MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	sec x = 5			
	cos x = 0.2	M1		
	x = 1.37, 4.91 AWRT	A1A1	3	
(b)	$\tan^2 x = 3\sec x + 9$			
	$\sec^2 x - 1 = 3\sec x + 9$	M1		for using $\sec^2 x = 1 + \tan^2 x$ OE
	$\sec^2 x - 3\sec x - 10 = 0$	A1	2	AG
(c)	$(\sec x - 5)(\sec x + 2) = 0$	M1		or use of formula (attempt)
	sec x = 5, –2	A1		
	cos x = 0.2, –0.5			
	x = 1.37, 4.91	B1F		any 2 correct or ft their 2 answers in (a)
	2.09, 4.19	A1	4	all 4 correct, no extras
	Total		**9**	
4(a)(i)		B1	1	$y = \lvert x \rvert$
(ii)		M1		2 branches mod graph $x > 0$ for $y = 0$
		A1	2	for 2, 4
(b)(i)	$x = 2x - 4$, $x = 4$	B1		
	$-x = 2x - 4$	M1		
	$x = \frac{4}{3}$	A1	3	OE one value only
	Alternative:			
	$x^2 = (2x-4)^2$	M1		
	$x = 4, \frac{4}{3}$	A1A1		
(ii)	$\frac{4}{3} < x < 4$	M1		$\frac{4}{3}, 4$ (ft) identified as extremes
		A1	2	CAO
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y = e^{2x} - 10e^{x} + 12x$			
(i)	$\frac{dy}{dx} = 2e^{2x} - 10e^{x} + 12$	B1 B1	2	$2e^{2x}$ remaining terms correct, no extras
(ii)	$\frac{d^2y}{dx^2} = 4e^{2x} - 10e^{x}$	B1F	1	ft 1 slip
(b)(i)	$2e^{2x} - 10e^{x} + 12 = 0$ $e^{2x} - 5e^{x} + 6 = 0$	B1	1	AG (be convinced)
(ii)	$z^2 - 5z + 6 = 0$	M1		use of $z = e^x$ oe
	$z = 2, 3$ $z = 2,\ e^x = 2$ $x = \ln 2$	M1		finding $e^x =$ their 2,3
	$z = 3,\ e^x = 3$ $x = \ln 3$	A1	3	all correct AG SC: verification $\ln 2$ (B1) $\ln 3$ (B1)
(iii)	$x = \ln 2$: $y = e^{2\ln 2} - 10e^{\ln 2} + 12\ln 2$ or $2^2 - 10 \times 2 + 12\ln 2$	M1		either substitution of their $x = \ln 2$ $(e^x = 2)$ or their $x = \ln 3$ $(e^x = 3)$
	$= 4 - 20 + 12\ln 2$ $= -16 + 12\ln 2$	A1		
	$x = \ln 3$: $y = e^{2\ln 3} - 10e^{\ln 3} + 12\ln 3$ $= 9 - 30 + 12\ln 3$ $= -21 + 12\ln 3$	A1	3	
(iv)	$x = \ln 2$: $\frac{d^2y}{dx^2} = 4e^{2\ln 2} - 10e^{\ln 2}$	M1		use of; in either of their $e^x = 2,3$ into their $\frac{d^2y}{dx^2}$
	$= 16 - 20 = -4$ $\therefore$ maximum	A1		CSO
	$x = \ln 3$: $\frac{d^2y}{dx^2} = 4e^{2\ln 3} - 10e^{\ln 3}$ $= 36 - 30 = 6$ $\therefore$ minimum	A1	3	CSO
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\therefore \int \ln x = 1(\ln 1.5 + \ln 2.5 + \ln 3.5 + \ln 4.5)$ $= 4.08$	M1 A1 A1	 3	use of 1.5, 2.5,… ; 3 or 4 correct x values AWFW 4 to 4.2 CAO
(b)(i)	$y = x \ln x$ $\frac{dy}{dx} = x \times \frac{1}{x} + \ln x$ $= \ln x + 1$	 M1 A1	 2	 use of product rule (only differentiating, 2 terms with + sign)
(ii)	$\int(\ln x + 1)\,dx = x \ln x$ $\int \ln x\,dx = x \ln x - x(+c)$	M1 A1	 2	OE; attempt at parts with $u = \ln x$
(iii)	$\int_1^5 \ln x\ dx = [x \ln x - x]_1^5$ $= (5\ln 5 - 5) - (1\ln 1 - 1)$ $5\ln 5 - 4$	 M1 A1	 2	 correct substitution of limits into their (ii) provided $\ln x$ is involved ISW
	Total		**9**	
7(a)	$z = \frac{\sin x}{\cos x}$ $\frac{dz}{dx} = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$ $= \frac{1}{\cos^2 x}$ $= \sec^2 x$	 M1 A1 A1	 3	 use of quotient rule $\left(\frac{\pm\cos^2 x \pm \sin^2 x}{\cos^2 x}\right)$ AG (be convinced)
(b)		M1 A1	 2	correct shape including asymptotic behaviour and symmetrical about $x = 0$ and $y > 0$ use of 1
(c)	$V = (k)\int \sec^2 x\,dx$ $= (k)[\tan x]_0^1$ $= 4.89$	M1 A1 A1	 3	 CAO
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$f(x)=2e^{3x}-1$			
	Range: $f(x)>-1$ (or $y>-1$ or $f>-1$)	M1		for -1 only
		A1	2	exactly correct
(b)	$y=2e^{3x}-1$			
	$x=2e^{3y}-1$	M1		$x\leftrightarrow y$
	$2e^{3y}=x+1$			
	$e^{3y}=\frac{x+1}{2}$	M1		attempt to isolate
	$y=\frac{1}{3}\ln\left(\frac{x+1}{2}\right)$	A1	3	all correct with no error AG (be convinced)
(c)	$f'^{-1}(x)=\frac{1}{3}\left(\frac{2}{x+1}\right)\times\frac{1}{2}$ OE	M1		for differentiation of ln; $\frac{k}{\text{their}(x\pm1)}$
		A1		for $\frac{1}{2}$
		A1		all correct
	$x=0$			
	$f'^{-1}(x)=\frac{1}{3}$	A1	4	CSO
	Alternative			
	$f^{-1}(x)=\frac{1}{3}\ln(x+1)-\frac{1}{3}\ln 2$	M1A1		
	$f'^{-1}(x)=\frac{1}{3(x+1)}$	A1		
	$f'^{-1}(0)=\frac{1}{3}$	A1		CSO
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$x=\frac{1}{2}$ $y=\frac{\pi}{2}$ $\left(\text{or } 1.57, \sin^{-1}1\right)$	B1	1	ignore 90°
(b)(i)	$y=\sin^{-1}2x$ $\sin y = 2x$ and $\frac{1}{2}\sin y = x$	B1	1	AG (be convinced)
(ii)	$\frac{dx}{dy}=\frac{1}{2}\cos y$	B1	1	
(c)	$\frac{dy}{dx}=\frac{2}{\cos y}$	M1A1		M1 for $\frac{k}{\cos y}$
	$\sin y = 2x$ and $\sin^2+\cos^2=1$ $\cos y=\sqrt{1-4x^2}$	M1		use of to get $\cos y$ or $\cos^2 y$
	$\frac{dy}{dx}=\frac{2}{\sqrt{1-4x^2}}$	A1	4	AG; condone omission of proof of sign
	Total		**7**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2007 examination - January series

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Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Jan 07

MPC3

Q	Solution	Marks	Total	Comments
1	$x =$ 1.5, 2.5, 3.5, 4.5	M1		Method
		A1		x values
	$y_1 = 0.7115$ 0.712 $y_2 = 0.5218$ 0.522 $y_3 = 0.4439$ 0.444 $y_4 = 0.3993$ 0.399 } AWRT	A1		3 correct y's
	$A = 1 \times (y_1 + y_2 + y_3 + y_4)$			
	$= 2.08$	A1	4	
	Total		**4**	
2	Stretch (I)			
	SF $\frac{1}{3}$ (II)	M1		For I + (II or III)
	Parallel to x – axis (III)	A1		All correct
	Translate	E1		Allow translation
	$\begin{pmatrix}0\\1\end{pmatrix}$	B1	4	Correct vector or description
	Total		**4**	
3(a)	$f(x) \leq 3$	M1A1	2	M1 for $f < 3$, $x \leq 3$ Condone y, f, range
(b)(i)	$y = \dfrac{2}{x+1}$			
	$x + 1 = \dfrac{2}{y}$	M1		Attempt to obtain x as a function of y or y as a function of x
	$x = \dfrac{2}{y} - 1$	M1		$x \leftrightarrow y$ at any stage
	$y/g^{-1}(x) = \dfrac{2}{x} - 1 = \dfrac{2-x}{x}$	A1	3	Any correct form
(ii)	$(g^{-1}(x)) \neq -1$	B1	1	
(c)(i)	$h(x) = \dfrac{2}{3 - x^2 + 1}$	M1		
	$= \dfrac{2}{4 - x^2} = \dfrac{2}{(2-x)(2+x)}$	A1	2	
(ii)	$(x \in \mathbb{R})$, $x \neq +2$, $x \neq -2$	B1	1	Condone omit 'x is real' Allow $x^2 \neq 4$
	Total		**9**	

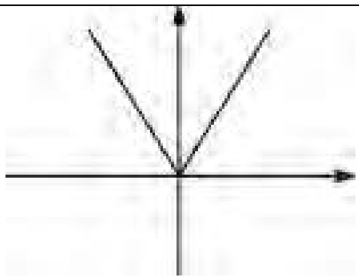
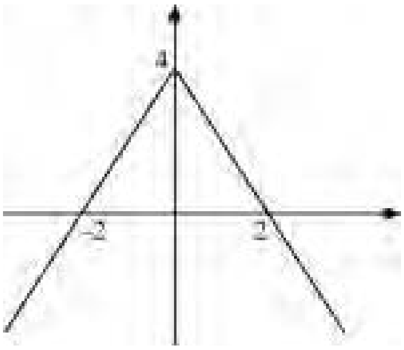
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\int x\sin x\,dx \quad u=x$			
	$\frac{dv}{dx}=\sin x$	M1		For differentiating one term and integrating other
	$\frac{du}{dx}=1 \quad v=-\cos x$			
	$\int = -x\cos x-\int -\cos x(dx)$	m1		For correctly substituting their terms into parts formula
		A1		
	$=-x\cos x+\sin x\ (+c)$	A1	4	CSO
(b)	$u=x^2+5$ $du=2x\,dx$			
	$\int=\int\frac{1}{2}u^{\frac{1}{2}}(du)$	M1		$\int ku^{\frac{1}{2}}(du)$ condone omission of du but M0 if dx
		A1		$k=\frac{1}{2}$ OE
	$=\frac{u^{\frac{3}{2}}}{3}$	A1√		Ft $\int ku^{\frac{1}{2}}du$
	$=\frac{1}{3}\sqrt{(x^2+5)^3}\ (+c)$	A1	4	CSO **SC** $\frac{2}{6}\sqrt{(x^2+5)^3}$ with no working B3
(c)	$y=x^2-9$ $x^2=y+9$			
	$V=\pi\int x^2\,dy$	B1		Must have π and x^2, condone omission of dy, but B0 if dx
	$=\pi\int(y+9)\,dy$			
	$=(\pi)\left[\frac{y^2}{2}+9y\right]_1^2$ or $(\pi)\left[\frac{(y+9)^2}{2}\right]_1^2$	M1		$\int$"their x^2"dy integrated; Limits 2 and 1 substituted in correct order including – sign } π not necessary
	$=(\pi)\left[20-9\frac{1}{2}\right]$	m1		
	$=10\frac{1}{2}\pi$	A1	4	CSO
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$2\left(\text{cosec}^2 x - 1\right) + 5\ \text{cosec}\, x = 10$	M1		
	$2\,\text{cosec}^2 x - 2 + 5\,\text{cosec}\, x - 10 = 0$			
	$2\ \text{cosec}^2 x + 5\ \text{cosec}\ x - 12 = 0$	A1	2	**AG**
(ii)	$(2\,\text{cosec}\, x - 3)(\text{cosec}\, x + 4) = 0$	M1		Attempt to solve
	$\text{cosec}\, x = \frac{3}{2}$ or -4	A1		Condone answers with no method shown
	$\sin x = \frac{2}{3}$ or $-\frac{1}{4}$	A1	3	**AG**
(b)	$(\theta - 0.1) = 0.73,\ 2.41,\ -0.25,\ -2.89$ AWRT	B1		2 correct values, may be implied later $(41.8, 138.2, -165.5, -14.5)$
	$\theta = 0.83,\ 2.51,\ -0.15,\ -2.79$ AWRT	B1		2 correct answers
		B1	3	+ 2 correct answers and no extra within range
	Total		**8**	
6(a)(i)	$y = \left(4x^2 + 3x + 2\right)^{10}$			
	$\frac{dy}{dx} = 10\left(4x^2 + 3x + 2\right)^9 (8x + 3)$	M1 A1	2	For $f(x)(\quad)^9$ where $f(x) \neq k$ and is linear
(ii)	$y = x^2 \tan x$	M1		Product rule
	$\frac{dy}{dx} = x^2 \sec^2 x + 2x \tan x$	A1	2	
(b)(i)	$x = 2y^3 + \ln y$			
	$\frac{dx}{dy} = 6y^2 + \frac{1}{y}$	B1	1	
(ii)	At $(2,1)$			
	$\frac{dx}{dy} = 6 + 1 = 7$	M1		
	$\frac{dy}{dx} = \frac{1}{7}$	A1√		May be implied
	$(y - 1) = \frac{1}{7}(x - 2)$	A1	3	OE
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)		B1	1	
(b)		M1 A1 A1	 3	Shape inverted V in all four quadrants Symmetrical about y axis Coordinates
(c)	$4-\lvert 2x \rvert = x$			
	$4-2x=x \quad x=\frac{4}{3}$	M1 A1		Attempt to solve
	$4+2x=x \quad x=-4$	A1	3	And no others
(d)	$-4<x<\frac{4}{3}$	M1 A1	 2	Either correct Other solution and no extras **SC** $-4 \le x \le \frac{4}{3}$ B1
	Total		**9**	
8(a)	$A(-1,\pi)$	B1		
	$B\left(0,\frac{\pi}{2}\right)$	B1	2	
(b)	$\cos^{-1}x-3x-1=0$			
	$f(0.1)=0.17$ allow 0.2, 0.1	M1		Or comparing 'sides'
	$f(0.2)=-0.23$ allow -0.2			
	Change of sign $\therefore$ root	A1	2	
(c)	$x_1=0.1$	M1		
	$x_2=0.1569=0.157$	A1		
	$x_3=0.1378=0.138$			
	$x_4=0.144$	A1	3	
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9(a)(i)	$\int\left(4-e^{2x}\right)dx$			
	$=4x-\frac{1}{2}e^{2x}\ (+c)$	B1 B1	2	$4x$ $-\frac{1}{2}e^{2x}$
(ii)	$\int_0^{\ln 2}=\left[4x-\frac{1}{2}e^{2x}\right]_0^{\ln 2}$			
	$=\left[4\ln 2-\frac{1}{2}e^{2\ln 2}\right]-\left[(0)-\frac{1}{2}\left(e^0\right)\right]$	M1		Substitute both ln 2 and 0 correctly into an integrated expression
	$=4\ln 2-2+\frac{1}{2}$			Convincing
	$=4\ln 2-\frac{3}{2}$	A1	2	**AG**
(b)(i)	$x=0$ $y=4-1=\underline{\underline{3}}$	B1	1	
(ii)	At B, $y=0$ $4-e^{2x}=0$	M1		Or reverse argument
	$e^{2x}=4$ $x=\ln 2$	A1	2	**AG**
(c)	$\frac{dy}{dx}=-2e^{2x}$	B1		
	$x=\ln 2$, Gradient $=-2e^{2\ln 2}$ $=-8$	M1		$x=\ln 2$ into ke^{2x}
	Gradient normal $=\frac{1}{8}=\frac{1}{2e^{2\ln 2}}$	A1		OE
	Equation $y=\frac{1}{8}x-\frac{1}{8}\ln 2$	A1	4	OE
(d)	When $x=0$ $y=-\frac{1}{8}\ln 2$	M1		Attempt to integrate their line and substitute $x=0, \ln 2$
	Area Δ $=\frac{1}{16}(\ln 2)^2$ condone – ve sign $=0.03$	A1√		$\frac{1}{2}(\text{their } y)\times\ln 2$
	Total area $=4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2=1.30$ AWRT	A1	3	CSO
	Total		**14**	
	Total		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2007 examination - June series

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MPC3

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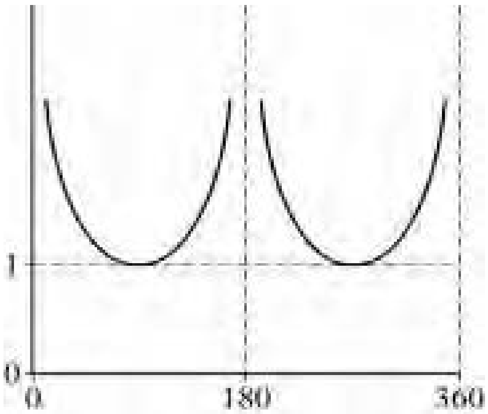
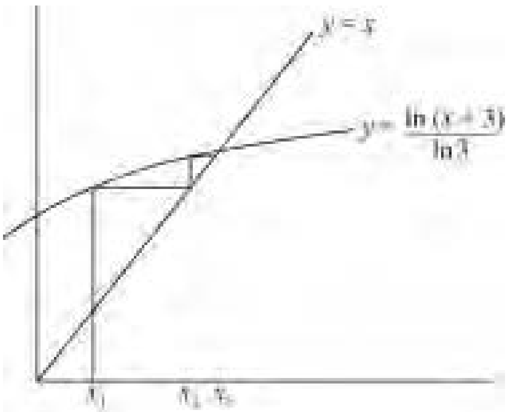
1(a)	$y = \ln x$			penalise + c once on 1(a) or 2(a)
	$\frac{dy}{dx} = \frac{1}{x}$	B1	1	
(b)	$y = (x+1)\ln x$			
	$\frac{dy}{dx} = (x+1)\times\frac{1}{x} + \ln x$	M1 A1	2	product rule
(c)	$y = (x+1)\ln x$			
	$\frac{dy}{dx} = \frac{1}{x} + 1 + \ln x$			
	$x = 1$: $\frac{dy}{dx} = 1 + 1 = 2$	M1		substitute $x = 1$ into their $\frac{dy}{dx}$
	Grad normal $= -\frac{1}{2}$	M1 A1		use of $m_1 m_2 = -1$ CSO
	$y = -\frac{1}{2}(x-1)$	A1	4	OE
	Total		**7**	
2(a)	$4(x-1)^3$ or in expanded form	B1	1	allow $-4(1-x)^3$
(b)	$V = 4\,(\pi)\int_2^4 (x-1)^3\,dx$	M1		$(\pi)\int y^2\,dx$
	$= \cancel{4}\pi\left[\frac{(x-1)^4}{\cancel{4}}\right]_2^4$	M1 m1		$k(x-1)^4\ (\pi)$ or in expanded form correct substitution of limits into $k(x-1)^4$
	$= \pi(81-1) = 80\pi$	A1	4	CAO
(c)	Translate	E1		
	$\begin{pmatrix}1\\0\end{pmatrix}$	B1		OE
	Stretch (I) SF 2 (II) // y axis (III)	M1 A1	4	for I and (II or III) for I and II and III
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	cosec $x = 2$			
	$\Rightarrow \sin x = \frac{1}{2}$	M1		30° scores M1 implied
	$x = 30,\ 150$	A1	2	and no extras in range
(b)(i)	1	B1	1	
(ii)		M1		all positive, 2 U shapes
		A1	2	minima consistent > 0, **not** intersecting with each other or y-axis
(c)	$x = 30, 150, 210, 330$	B1F		3 correct values from their (a), which must be $\theta, 180-\theta$
		B1	2	all correct and no extras in range
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	y x_0 1 3 x_1 1.25 3.948(2) x_2 1.5 5.196(2) x_3 1.75 6.838(5) x_4 2 9	B1 B1		x values PI (4 +) y values correct
	$A = \frac{1}{3} \times \frac{1}{4}(3 + 4 \times 3.9482 + 2 \times 5.1962$ $+ 4 \times 6.8385 + 9)$	M1		Simpson's rule
	$= 5.46$	A1	4	CAO
(b)(i)	$f(x) = 3^x - x - 3$ $\left.\begin{array}{l} f(0.5) = -1.77 \\ f(1.5) = 0.696 \end{array}\right\}$ change of sign ∴ root	M1A1	2	
(ii)	$3^x = x + 3$ $\ln 3^x = \ln(x+3)$	M1		correct use of logs
	$x \ln 3 = \ln(x+3)$ $x = \frac{\ln(x+3)}{\ln 3}$	A1	2	correct with no mistakes; AG
(iii)	$x_1 = 0.5$ $(x_2 = 1.14)$	M1		
	$x_3 = 1.29 = 1.3$	A1	2	CAO
(iv)	 	M1 A1	2	staircase x_2, x_3 correct and labelled on x-axis
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$f(x) \geq 0$ allow $y \geq 0$	M1		>0 or $f \geq 0$ or ≥ 0
		A1	2	
(b)(i)	$\sqrt{\frac{1}{x}-2}$	B1	1	
(ii)	$\frac{1}{x}-2=1$	M1		squaring their (b)(i) in an equation
	$\frac{1}{x}=3$ OE	A1		
	$x=\frac{1}{3}$	A1	3	CSO
(c)	$y=\sqrt{x-2}$			
	$y^2=x-2$	M1		attempt to isolate; condone 1 slip
	$x^2=y-2$	M1		reverse $x \Leftrightarrow y$
	$y=x^2+2$	A1	3	
	Total		**9**	
6(a)	$\int x e^{5x} dx$			
	$u=x \quad dv=e^{5x}$	M1		integrate one term, differentiate one term
	$du=1 \quad v=\frac{1}{5}e^{5x}$	A1		
	$\int=\frac{1}{5}xe^{5x}-\int\frac{1}{5}e^{5x}dx$	A1		
	$=\frac{1}{5}xe^{5x}-\frac{1}{25}e^{5x}(+c)$	A1	4	
(b)(i)	$u=x^{\frac{1}{2}}$			
	$du=\frac{1}{2}x^{-\frac{1}{2}}\ dx$	M1		
	$\int=\int\frac{1}{1+u}\times 2\,du$	A1	2	correct with no errors; AG
(ii)	$\int_1^9 dx=\int_1^3\frac{2}{1+u}du$	m1		correct limits used in correct expression, ignoring k
	$=\left[2\ln(1+u)\right]_1^3$	M1		for $k\ln(1+u)$
	$=2\ln 4-2\ln 2$ $(=\ln 4)$	A1	3	ISW OE
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$y=\left(x^2-3\right)e^x$			
	$\frac{dy}{dx}=\left(x^2-3\right)e^x+2xe^x$	M1 A1	2	product rule
(ii)	$\frac{d^2y}{dx^2}=\left(x^2-3\right)e^x+2xe^x+2xe^x+2e^x$	M1 A1	2	product rule from their $\frac{dy}{dx}$
(b)(i)	$\frac{dy}{dx}=0$			
	$\Rightarrow e^x\left(x^2+2x-3\right)=0$	M1		$e^x f(x)=0$ from $\frac{dy}{dx}=0$
	$e^x(x+3)(x-1)=0$	m1		attempt at factorising or use of formula
	$\therefore x=-3, 1$	A1		first correct solution
		A1	4	second correct solution, and no others SC No working shown: $x=-3$ B2, $x=1$ B2
(ii)	$x=-3$ $y''=-4e^x$ max (-0.2)	M1		Condone slip
	$x=1$ $y''=4e^x$ min (10.9)	A1	2	
	Total		**10**	
8(a)	$\tan x$ $(+c)$	B1	1	
(b)	$f(x)=\frac{\cos x}{\sin x}$			
	$f'(x)=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$	M1 A1		quotient rule $\frac{\pm\sin^2 x\pm\cos^2 x}{\sin^2 x}$
	$=\frac{-1}{\sin^2 x}$	A1		use of $\sin^2 x+\cos^2 x=1$
	$=-\text{cosec}^2 x$	A1	4	AG CSO **Special cases** $f(x)=\frac{\cot x}{1}$ $f'(x)=\frac{1\times-\text{cosec}^2 x-\cot x\times 0}{1^2}$ M1 $=-\text{cosec}^2 x$ A1 (max 2/4) **Or** $f(x)=\frac{1}{\tan x}$ $f'(x)=\frac{\tan x\times 0-1\times\sec^2 x}{\tan^2 x}$ M1 A1 $=\frac{-\sec^2 x}{\tan^2 x}$ $=\frac{-1}{\sin^2 x}=-\text{cosec}^2$ A1 (max 3/4)

MPC3 (cont)

Q	Solution	Marks	Total	Comments
(c)	LHS $= \tan^2 x + \cot^2 x + 2\tan x \cot x$	M1		expanding
	$= \tan^2 x + 1 + \cot^2 x + 1$	M1		correct use of trig identities
	$= \sec^2 x + \operatorname{cosec}^2 x$ $=$ RHS	A1	3	CSO
(d)	$\int (\tan x + \cot x)^2 \, dx = \int \sec^2 x + \operatorname{cosec}^2 x \, dx$	M1		use of identity
	$= [\tan x - \cot x]_{0.5}^{1}$	M1 A1		$\pm \tan x \pm \cot x$ OE
	$= 0.9153 - -1.2842$			
	$= 2.2$	A1	4	AWRT
	Total		**12**	
	TOTAL		**75**	

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Mathematics 6360

MPC3 Pure Core 3

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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

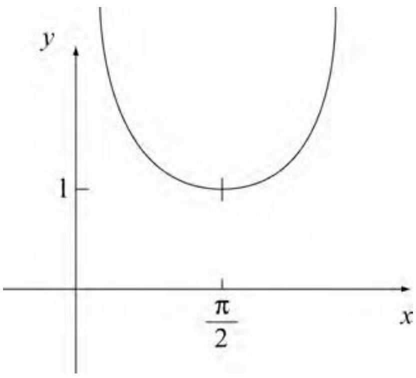
MPC3

Q	Solution	Marks	Total	Comments
1(a)(i)	$y=\left(2x^2-5x+1\right)^{20}$			
	$\frac{dy}{dx}=20\left(2x^2-5x+1\right)^{19}(4x-5)$ OE	M1 A1	2	chain rule $20(\quad)^{19}\,f(x)$ with no further incorrect working
(ii)	$y=x\cos x$			
	$\frac{dy}{dx}=-x\sin x+\cos x$	M1 A1	2	product rule $\pm x\sin x\pm\cos x$ CSO
(b)	$y=\frac{x^3}{x-2}$			
	$\frac{dy}{dx}=\frac{(x-2)3x^2-x^3\times 1}{(x-2)^2}$	M1		quotient rule $\frac{\pm vu'\pm uv'}{(x-2)^2}$
		A1		condone missing brackets
	$=\frac{3x^3-6x^2-x^3}{(x-2)^2}$			
	$=\frac{2x^2(x-3)}{(x-2)^2}$	A1	3	CSO
	Total		**7**	
2(a)	$\cot x=2\Rightarrow\tan x=0.5$	M1		
	$x=0.46,\ 3.61$	A1	2	AWRT; no others within range
(b)	$\text{cosec}^2 x=\frac{3\cot x+4}{2}$			
	$2\left(1+\cot^2 x\right)=3\cot x+4$	M1		Correct use of $\text{cosec}^2 x=1+\cot^2 x$
	$\left(2\cot^2 x-3\cot x+2-4=0\right)$			
	$2\cot^2 x-3\cot x-2=0$	A1	2	AG; correct with no slips from line with no fractions
(c)	$(2\cot x+1)(\cot x-2)(=0)$	M1		Attempt to solve
	$\cot x=-\frac{1}{2},\ 2$	A1		
	$\tan x=-2,\ 0.5$			
	$x=0.46,\ 3.61,\ 2.03,\ 5.18$	B1 B1	4	2 correct Allow 3.6(0) 4 correct (with no extras in range) AWRT **SC** Degrees 26.57, 206.57 116.57, 296.57 } B1 for 2 correct
	Total		**8**	

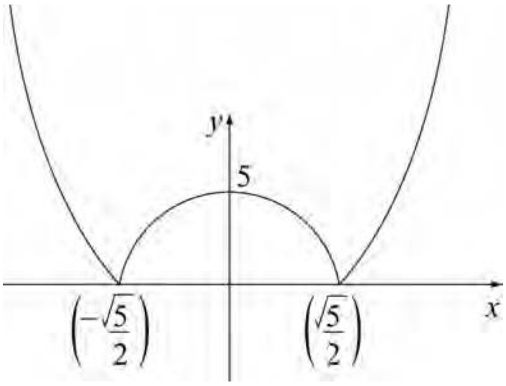
MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$x+(1+3x)^{\frac{1}{4}}=0$ $\mathrm{f}(-0.32)=0.1$ $\mathrm{f}(-0.33)=-0.01$ Change of sign $\therefore\ -0.33<x<-0.32$	M1 A1	 2	AWRT; allow + ve, –ve
(b)	$x=-(1+3x)^{\frac{1}{4}}$ $x^4=1+3x$ $\frac{x^4-1}{3}=x$	M1 A1	 2	Attempt to isolate x^4 AG
(c)	$x_1=-0.3$ $(x_2=-0.331)$ AWRT $(x_3=-0.329)$ AWRT $x_4=-0.329$	M1 A1 A1	 3	
	Total		**7**	
4(a)	all (real) values	B1	1	No x in answer, unless f (x)
(b)(i)	$\mathrm{fg}(x)=\left(\frac{1}{x-3}\right)^3$	B1	1	ISW
(ii)	$\left(\frac{1}{x-3}\right)^3=64$ $\frac{1}{x-3}=4$ $x-3=\frac{1}{4}$ $x=3\frac{1}{4}$	 M1 M1 A1	 3	 $\sqrt[3]{\ }$ Invert
(c)(i)	$y=\frac{1}{x-3}$ $x=\frac{1}{y-3}$ $x(y-3)=1$ $xy-3x=1$ $y=\frac{1+3x}{x}=\mathrm{g}^{-1}(x)$ or $\frac{1}{x}+3$	 M1 M1 A1	 3	 Swap x and y attempt to isolate
(ii)	(real values) $(\mathrm{g}^{-1}(x))\neq 3$	B1	1	
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$y = 2x^2 - 8x + 3$			
	$\left(\frac{dy}{dx} =\right) 4x - 8$	B1	1	
(ii)	$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\, dx$			
	$= \frac{1}{4}\left[\ln\left\|2x^2 - 8x + 3\right\|\right]_4^6$	M1A1		M1 for $k \ln\left(2x^2 - 8x + 3\right)$; allow $k \ln u$
	$= \frac{1}{4}\left[\ln 27 - \ln 3\right]$	m1		Correct substitution into $k \ln\left(2x^2 - 8x + 3\right)$ or 3, 27 into $k \ln u$
	$= \frac{1}{4}\ln 9$			
	$= \frac{1}{2}\ln 3$	A1	4	
(b)	$\int x\sqrt{(3x-1)}\, dx$			
	$u = 3x - 1 \quad du = 3dx$	B1		OE
	$\int = \left(\frac{1}{9}\right)\int\left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)(du)$	M1		$\int$ 2 terms in u with rational indices
	$= \left(\frac{1}{9}\right)\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} (+c)\right]$	A1F		Must be 2 terms with correct indices $\left(\text{only ft for } x = \frac{u-1}{3}\right)$
	$= \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + \boldsymbol{c}$	A1	4	CSO OE
	Total		**9**	
6(a)		M1 A1	 2	Correct shape Vertex
(b)	x / y: 0.15 / 6.692 0.25 / 4.042 0.35 / 2.916 0.45 / 2.299	M1 B1		Correct x values ≥ 3 correct y values
	$\int \simeq 0.1 \times \sum y \quad \left(\sum y = 15.949\right)$	B1		correct h used correctly
	= 1.59	A1	4	
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Stretch (I)			
	Scale factor $\frac{1}{2}$ (II)	M1		I + (II or III)
	parallel to x-axis (III)	A1		All correct
	(Or scale factor 4 parallel to y-axis)			
	Translation	M1		
	$\begin{bmatrix} 0 \\ -5 \end{bmatrix}$ OE	A1	4	
	Alternatives			
	translate $\begin{pmatrix} 0 \\ -\frac{5}{4} \end{pmatrix}$, stretch sf 4 $\parallel$ y-axis			Mark translation first. Mark stretch as above, but relative to their translation.
	translate $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$, stretch sf $\frac{1}{2}$ $\parallel$ x-axis			
(b)		M1		Modulus graph symmetrical about y-axis
		A1		left of $-\frac{\sqrt{5}}{2}$ and right of $\frac{\sqrt{5}}{2}$
		A1	3	(0, 5), cusps drawn and no straight lines between cusps
(c)(i)	$4x^2 - 5 = 4$			
	$4x^2 = 9$			
	$x = \pm\frac{3}{2}$ OE	B1		
	$4x^2 - 5 = -4$	M1		$16x^4 - 40x^2 + 9 = 0$
	$4x^2 = 1$			
	$x = \pm\frac{1}{2}$	A1	3	
(ii)	$x \le -\frac{3}{2}$, $x \ge \frac{3}{2}$	B1F		2 correct statements
	$-\frac{1}{2} \le x$, $x \le \frac{1}{2}$	B1F	2	4 correct statements
				SC c(ii) 1 mark penalty for strict inequalities
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$e^{-2x} = 3$			
	$-2x = \ln 3$	M1		
	$x = -\frac{1}{2}\ln 3$	A1	2	OE ISW
(b)	$\int x e^{-2x}\ dx$			
	$u = x \quad \frac{dv}{dx} = e^{-2x}$			
	$\frac{du}{dx} = 1 \quad v = -\frac{1}{2}e^{-2x}$	M1		differentiating and integrating
	$\int = -\frac{1}{2}xe^{-2x} + \int \frac{1}{2}e^{-2x}\ (dx)$	m1		correct subs of their values into parts formula
		A1		
	$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$	A1	4	No further incorrect working
(c)(i)	$y = e^{-2x} + 6x$			
	$\frac{dy}{dx} = -2e^{-2x} + 6 = 0$	M1		$ke^{-2x} + 6 = 0$
	$\frac{dy}{dx} = 0 \Rightarrow -2\left(e^{-2x} - 3\right) = 0$			
	$x = -\frac{1}{2}\ln 3$	A1		OE
	$y = 3 + 6\left(-\frac{1}{2}\ln 3\right)$	M1		Correct substitute of their valid x
	$= 3 - 3\ln 3$	A1	4	OE ISW
(ii)	$\frac{d^2y}{dx^2} = 4e^{-2x} \begin{cases} = 12 \\ > 0 \end{cases}$	M1		Other methods need justification Allow error in $\frac{d^2y}{dx^2}$ or x-value, but not both
	$\therefore$ minimum	A1	2	
(iii)	$(V) = \pi\int_0^1 y^2 dx = (\pi)\int_{(0)}^{(1)} \left(e^{-2x} + 6x\right)^2 (dx)$	M1		Either
	$= (\pi)\int_{(0)}^{(1)} \left(e^{-4x} + 12xe^{-2x} + 36x^2\right) dx$	B1		Correct expansion
	$= (\pi)\left[-\frac{1}{4}e^{-4x} - 6xe^{-2x} - 3e^{-2x} + 12x^3\right]_{(0)}^{(1)}$	A1		3 correct terms; '–6','–3' correct or 12 × their (b)
		A1		All correct
	$= \pi\left[\left(-\frac{1}{4}e^{-4} - 9e^{-2} + 12\right) - \left(-\frac{1}{4} - 3\right)\right]$			
	$= \pi\left[15\frac{1}{4} - 9e^{-2} - \frac{1}{4}e^{-4}\right]$			
	$= 44.1$	B1	5	AWRT
	Total		**17**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\frac{dy}{dx} = 5(3x+1)^4 \times 3$	M1		$k(3x+1)^4$
	$= 15(3x+1)^4$	A1	2	with no further errors (w.n.f.e)
(b)	$\frac{dy}{dx} = \frac{3}{3x+1}$	M1		$\frac{k}{3x+1}$
		A1	2	w.n.f.e
(c)	$\frac{dy}{dx} =$			
	$(3x+1)^5 \times \frac{3}{3x+1} + \ln(3x+1) \times 15(3x+1)^4$	M1		product rule $uv' + u'v$ (from (a) and (b))
		A1		either term correct
		A1	3	CSO with no further errors
	$\left(\begin{array}{l} = (3x+1)^4 \left[3 + 15\ln(3x+1) \right] \\ = 3(3x+1)^4 \left[1 + 5\ln(3x+1) \right] \end{array} \right)$			
	Total		**7**	
2(a)	$x = \cos^{-1} \frac{1}{3}$	M1		PI
	$= 1.23,\ 5.05 \quad (0.39\pi,\ 1.61\pi)$	A1,A1	3	AWRT (-1 for each error in range)
				SC 70.53, 289.47 B1
(b)	$\sec^2 x - 1 = 2\sec x + 2$	M1		use of $\sec^2 x = 1 + \tan^2 x$
	$\sec^2 x - 2\sec x - 3 = 0$	A1	2	AG; CSO
(c)	$\sec^2 x - 2\sec x - 3 = 0$			
	$(\sec x - 3)(\sec x + 1) = 0$	M1		attempt to solve
	$\cos x = \frac{1}{3}$ or -1 o.e	A1		
	$x = 1.23,\ 5.05,$	B1f		(2 answers in range from (a)) AWRT
	$3.14 \quad (\pi)$	B1	4	all correct and no extras in range
				SC 70.53, 289.47, 180 B1
	Total		**9**	

(Extra +c penalised once throughout paper)

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{dy}{dx} = -x\,2\sin 2x + \cos 2x$	M1 A1	2	product rule $kx\sin 2x \pm \cos 2x$ no further incorrect working
(b)(i)	$-2\alpha\sin 2\alpha + \cos 2\alpha = 0$	M1		replacing $x = \alpha$ and writing equation equal to zero (at any line)
	$\left.\begin{array}{l}2\alpha\sin 2\alpha = \cos 2\alpha \\ 2\alpha\tan 2\alpha = 1\end{array}\right\}$ either $2\alpha\tan 2\alpha - 1 = 0$	A1	2	AG; CSO
(ii)	$\left.\begin{array}{l}f(0.4) = 0.2 \\ f(0.5) = -0.6\end{array}\right\}$ awrt o.e.	M1		(0.9's unsubstantiated scores M0)
	Change of sign $\therefore 0.4 < \alpha < 0.5$	A1	2	
(iii)	$2x\tan 2x = 1$ $\left.\begin{array}{l}\tan 2x = \frac{1}{2x} \\ 2x = \tan^{-1}\left(\frac{1}{2x}\right)\end{array}\right\}$ either $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$	B1	1	AG; CSO
(iv)	$x_1 = 0.4$ $x_2 = 0.4480...$	M1		$x_2 = 25.7$
	$x_3 = 0.4200...$ $= 0.42$	A1	2	
(c)	$y = x\cos 2x$ $\left.\begin{array}{ll}u = x & du = 1 \\ dv = \cos 2x & v = \frac{\sin 2x}{2}\end{array}\right\}$	M1		$\left.\begin{array}{l}\text{differentiate one term} \\ \text{integrate one term}\end{array}\right\}$ must be $k\sin 2x$
		m1		correct substitution of their values into parts formula using $u = x$
	$\int = \frac{x\sin 2x}{2} - \int\frac{\sin 2x}{2}\,(dx)$			
	$= \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_{(0)}^{(0.5)}$	A1		
	$= \left(\frac{\sin 1}{4} + \frac{\cos 1}{4}\right) - \left(\frac{\cos 0}{4}\right)$	m1		correctly substituting values from previous 2 method marks
	$= 0.0954$	A1	5	AWRT
	Total		**14**	

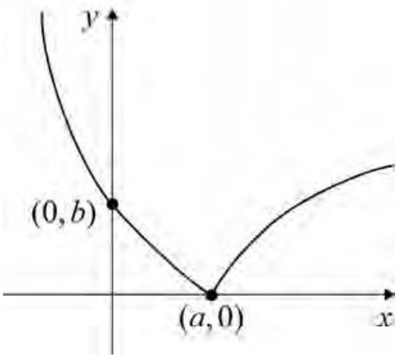
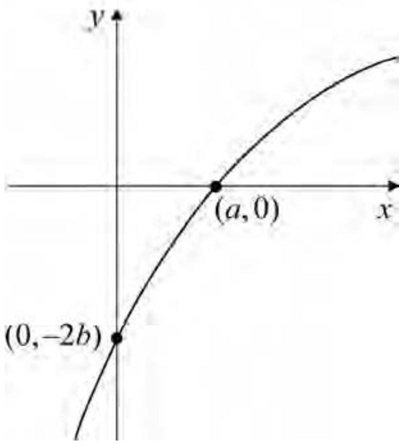
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$f(x) \geqslant 0$	B1	1	allow $f \geqslant 0,\ y \geqslant 0,\ \geqslant 0$
(b)(i)	$y = \dfrac{1}{2x-3}$			
	$x = \dfrac{1}{2y-3}$	M1		swap x and y
	$x(2y-3) = 1$			
	$2xy - 3x = 1$			
	$2xy = 1 + 3x$	M1		attempt to isolate
	$y = \dfrac{1+3x}{2x} = g^{-1}(x)$ o.e.	A1	3	w.n.f.e
(ii)	$\left(g^{-1}(x)\right) \neq \dfrac{3}{2}$	B1	1	
(c)	$\left(\dfrac{1}{2x-3}\right)^2 = 9$	B1		
	$2x - 3 = \pm\dfrac{1}{3}$	M1		square root and invert (condone missing $\pm$) **alternative:** attempt to solve a quadratic that comes from $4x^2 - 12 + 9 = \dfrac{1}{9}$ o.e.
	$x = \dfrac{5}{3}, \dfrac{4}{3}$ o.e.	A1	3	
	Total		**8**	

Alternative

4(b)(i)	$x \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow \boxed{\text{divide into1}} \rightarrow y$ $\dfrac{1}{2y} + \dfrac{3}{2} \leftarrow \boxed{\div 2} \leftarrow \boxed{+3} \leftarrow \boxed{\text{divide into1}} \leftarrow y$ $\dfrac{1}{y} + 3$ M1			

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)		B1		shape
		B1	2	coordinates
(ii)		B1		shape
		B1	2	coordinates
(b)(i)	Translation	M1		**OR** I stretch M1 I + (II or III) II SF 4 III ∥ y-axis A1 (I + II + III) Translation M1 $\begin{bmatrix}-1\\-2\end{bmatrix}$ A1 B1 All correct A1
	$\begin{bmatrix}-1\\0\end{bmatrix}$	A1		
	Stretch I SF 4 II ∥ y-axis III	M1		$\text{I}+(\text{II or III})$
		A1		I + II + III
	Translation $\begin{bmatrix}0\\-2\end{bmatrix}$	B1		both
	All correct and no mistakes on order etc	A1	6	
	Alternative:			
	$y=4\ln(x+1)-2=4\left[\ln(x+1)-\frac{1}{2}\right]$	(B1)		
	Translation	(M1)		
	$\begin{bmatrix}-1\\-\frac{1}{2}\end{bmatrix}$	(A1)		
	Stretch I SF 4 II ∥ y-axis III	(M1)		$\text{I}+(\text{II or III})$
		(A1)		I + II + III
	All correct and no mistakes on order etc	(A1)	(6)	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(b)(ii)	$y=4\ln(x+1)-2$			
	$x=0 \quad y=-2$	B1		
	$y=0$			
	$4\ln(x+1)=2$			
	$\ln(x+1)=\frac{1}{2}$	M1		isolate $\ln(x+1)=$ or $(x+1)^4$
	$x+1=e^{\frac{1}{2}}$	A1		$x+1=e^k$
	$x=e^{\frac{1}{2}}-1$ o.e.	A1	4	CSO isw
	Total		**14**	
6(a)	$y=\left(e^{3x}+1\right)^{\frac{1}{2}}$			
	$\frac{dy}{dx}=\frac{1}{2}\left(e^{3x}+1\right)^{-\frac{1}{2}}\times 3e^{3x}$	M1		$\frac{1}{2}\left(e^{3x}+1\right)^{-\frac{1}{2}}$
		A1		e^{3x}
		A1		$\frac{3}{2}$ (allow $\frac{1}{2}\times 3$) w.n.f.e
	$x=\ln 2:$			
	$\frac{dy}{dx}=\frac{3}{2}\left(e^{\ln 8}+1\right)^{-\frac{1}{2}}\times e^{\ln 8}$	M1		correct substitution into their $\frac{dy}{dx}$ (must use ln8 or $\ln 2^3$)
	$=\frac{3}{2}\times\frac{1}{3}\times 8$			
	$=4$	A1	5	
(b)	x : 0.25, 0.75, 1.25, 1.75 y : 1.765(5), 3.238(5), 6.597(1), 13.84(1)	B1		correct x values
		B1		3 or 4 correct y values 4 s.f. or better
	$\int = 0.5\times\sum y$ P.I	M1		
	$= 12.7$	A1	4	sc 12.7 with no working $\frac{2}{4}$
(c)	$v=\pi\int y^2 dx$			
	$=(\pi)\int\left(e^{3x}+1\right)(dx)$	M1		
	$=(\pi)\left[\frac{1}{3}e^{3x}+x\right]_{(0)}^{(2)}$	A1		$ke^{3x}+x$
	$=(\pi)\left[\left(\frac{1}{3}e^{6}+2\right)-\left(\frac{1}{3}e^{0}+0\right)\right]$	m1		correct substitution into f ($\int e^{3x}$)
	$=\pi\left[\frac{1}{3}e^{6}+\frac{5}{3}\right]$	A1	4	CSO
	$\left(=\frac{\pi}{3}(e^{6}+5)\right)$			
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$y = \dfrac{\sin\theta}{\cos\theta}$			
	$\dfrac{dy}{d\theta} = \dfrac{\cos\theta\cos\theta - \sin\theta(-\sin\theta)}{\cos^2\theta}$	M1 A1		$\dfrac{\pm\cos^2\theta \pm \sin^2\theta}{\cos^2\theta}$
	$= \dfrac{1}{\cos^2\theta}$ o.e.			$(1+\tan^2\theta)$
	$= \sec^2\theta$	A1	3	AG; CSO
(b)	$x = \sin\theta$ $x^2 = \sin^2\theta$ $\cos^2\theta = 1 - x^2$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $= \dfrac{x}{\sqrt{1-x^2}}$ **OR** LHS = $\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta$ AG	M1 A1	2	use of $\cos^2\theta + x^2 = 1$ AG; CSO
(c)	$\displaystyle\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx$			
	$x = \sin\theta$ $dx = \cos\theta\, d\theta$ o.e.	M1		$\dfrac{dx}{d\theta} = \pm\cos\theta$
	$\displaystyle\int = \int \frac{\cos\theta\,(d\theta)}{(1-\sin^2\theta)^{\frac{3}{2}}}$	m1		all in terms of θ
	$\displaystyle = \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}}\,(d\theta)$	A1		
	$\displaystyle = \int \sec^2\theta\,(d\theta)$	A1		
	$= \tan\theta$			
	$= \dfrac{x}{\sqrt{1-x^2}}\ (+c)$	A1	5	CSO including $d\theta$'s
	Total		**10**	
	TOTAL		**75**	

Alternative

7(a)	$y = \dfrac{\tan\theta}{1}$			
	$\dfrac{dy}{d\theta} = \dfrac{1\sec^2\theta - 0}{1^2}$	M1 A1		
	$= \sec^2\theta$	A1		

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General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2009 examination - January series

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Set and published by the Assessment and Qualifications Alliance.

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

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M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

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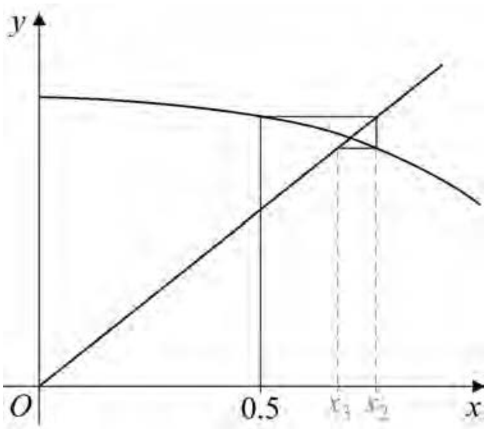
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1	x y			
	1 0.5	B1		x values and no extra values
	3 0.366(0)			
	5 0.309(0)	B1		4+ correct y values or $\frac{1}{1+\sqrt{3}}$ etc
	7 0.274(3)			
	9 0.25			
	$\int =$			
	$\frac{1}{3}\times 2\times\left[\begin{array}{l}(0.5+0.25)+\\ 4(0.3660+0.2743)+2(0.3090)\end{array}\right]$	M1		Correct application of Simpson's rule for their x values (x odd)
	$= 2.62$	A1	4	CSO must be 3sf
	Total		**4**	
2	$V=(\pi)\int y^2\,dx$			
	$=(\pi)\int (x-2)^5\,dx$	M1		
	$=(\pi)\left[\frac{(x-2)^6}{6}\right]_3^4$	A1		limits not required
	$=(\pi)\left(\frac{2^6}{6}-\frac{1}{6}\right)$	m1		correct substitution into $(\pi)k(x-2)^6$
	$=10.5\pi$	A1	4	allow equivalent fraction $\left(\frac{63}{6}\pi \text{ etc}\right)$ (AWRT 10.5 or 10.5π m1, A0)
	Total		**4**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$f(x) = x^3 + 5x - 4$			
	$f(0.5) = -1.375$ $f(1) = 2$	M1		Condone f (0.5) rounding to -1.4
	Change of sign $\therefore 0.5 < \alpha < 1$	A1	2	Both statements needed
(b)	$x^3 + 5x - 4 = 0$			
	$5x = 4 - x^3$			Must be seen
	$x = \frac{1}{5}(4 - x^3)$	B1	1	AG
(c)	$x_1 = 0.5$			
	$(x_2 = 0.775)$ $\left(= \frac{31}{40}\right)$	M1		For x_2 or x_3 $= (2\text{ sf})$
	$x_3 = 0.707$	A1	2	
(d)		M1		From 0.5 vertical to curve then horizontal to line
		A1	2	CAO
	Total		**7**	

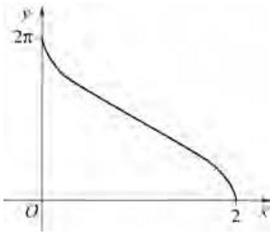
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\sec x = \frac{3}{2}$ $\cos x = \frac{2}{3}$ $x = 48, 312$ (Condone answers rounding to)	 B1 B1	 2	 1 correct 2 correct and no extras in interval
(b)	$2\tan^2 x = 10 - 5\sec x$ $2\left(\sec^2 x - 1\right) = 10 - 5\sec x$ $2\sec^2 x + 5\sec x - 12 (= 0)$ $(2\sec x - 3)(\sec x + 4)(= 0)$ $\left.\begin{array}{l}\sec x = \frac{3}{2}, -4 \\ \cos x = \frac{2}{3}, -\frac{1}{4}\end{array}\right\}$ either of these $x = 48, 312, 104, 256$	 M1 A1 m1 A1 B1 B1	 6	 Use of trig identity correctly Attempt to solve or factorise 1 slip using formula AWRT 3 correct condone 105 or 255 All correct and no extras in interval
	Alternative: $\left.\begin{array}{l}\frac{2\sin^2 x}{\cos^2 x} = 10 - \frac{5}{\cos x} \\ 2\sin^2 x = 10\cos^2 x - 5\cos x \\ 2 - 2\cos^2 x = 10\cos^2 x - 5\cos x\end{array}\right\}$ $12\cos^2 x - 5\cos x - 2 = 0$ then rest of scheme as above	 (M1) (A1)		
	Total		**8**	
5(a)	$f(x) \le 2, \quad f \le 2, \quad y \le 2$	B2	2	$\left.\begin{array}{l}\le 2, f(x) < 2, x \le 2 \\ y < 2, \ f < 2\end{array}\right\}$ B1
(b)	$f(x)$ is not one to one	E1	1	Allow many to one or numerical example
(c)(i)	$fg(x) = 2 - \left(\frac{1}{x-4}\right)^4$	B1	1	
(ii)	$2 - \left(\frac{1}{x-4}\right)^4 = -14$ $16 = \left(\frac{1}{x-4}\right)^4$ $\left.\begin{array}{l}(x-4)^4 = \frac{1}{16} \\ x - 4 = \pm\frac{1}{2}\end{array}\right\}$ $x = 4\frac{1}{2}, 3\frac{1}{2}$	 M1 M1 A1	 3	 Correct handling of fourth root Must have $\pm$ Correct handling of reciprocal
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$y = e^{2x}\left(x^2 - 4x - 2\right)$			
	$\frac{dy}{dx} = e^{2x}\left(2x - 4\right)$	M1		Product rule; allow 1 slip
	$+\left(x^2 - 4x - 2\right)2e^{2x}$	A1		
	$\frac{dy}{dx} = e^{2x}\left(2x - 4 + 2x^2 - 8x - 4\right)$	M1		Factorising $e^{2x}\ \left(ax^2 + 6x + 0\right)$
	$e^{2x}\left(2x^2 - 6x - 8\right)$	A1		or $x^2 - 3x - 4 = 0$
	$e^{2x} \neq 0$			
	$\left(x - 4\right)\left(x + 1\right) = 0$	m1		Solving 3 term quadratic Dependent on both M marks
	$x = 4, -1$	A1	6	And no extras eg $x = 0$
(b)(i)	$\frac{d^2y}{dx^2} = e^{2x}.2 + \left(2x - 4\right)2e^{2x}$	M1		Product rule from their $\frac{dy}{dx}$ in form
	$+\left(x^2 - 4x - 2\right)4e^{2x} + 2e^{2x}\left(2x - 4\right)$			e^{2x} (quadratic) $\quad e^{2x}\left(4x^2 - 8x - 22\right)$
		A1	2	
	Or			
	$\frac{d^2y}{dx^2} = e^{2x}\left(4x - 6\right) + \left(2x^2 - 6x - 8\right)2e^{2x}$	M1 A1		
(ii)	$x = 4 : y'' = e^{8}\left(10\right) > 0 \therefore$ MIN	M1		Their 2 x's in their $\frac{d^2y}{dx^2}$ only of form e^{2x} (quadratic)
	$x = -1 : y'' = e^{-2}\left(-10\right) < 0 \therefore$ MAX	A1	2	CSO Both correct Allow values either side of y or y'
	Total		**10**	
7(a)	$3e^{x} = 4$			
	$e^{x} = \frac{4}{3}$	M1		
	$x = \ln\frac{4}{3}$	A1	2	
(b)(i)	$3e^{x} + 20e^{-x} = 19$			
	$3y + \frac{20}{y} = 19$ or $3e^{2x} + 20 = 19e^{x}$			
	$3y^2 - 19y + 20 = 0$	B1	1	AG
(ii)	$\left(3y - 4\right)\left(y - 5\right) = 0$			
	$y = \frac{4}{3}, 5$	B1		
	$\therefore x = \ln\frac{4}{3}, \ln 5$	M1 A1	3	ln (their + ve y's)
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$P\,(-1,\ \pi)$	B1		Condone $(-1,\ 180°)$
	$Q\,(1,\ 0)$	B1	2	
(b)	Translate	E1		
	$\begin{bmatrix}1\\0\end{bmatrix}$	B1		or equivalent in words
	Stretch SF 2 $/\!/$ y-axis	M1		Stretch + one other correct
		A1	4	all correct
(c)		B1		Correct shape in 1st quadrant
		B1	2	2π and 2 marked correctly
(d)(i)	$\frac{y}{2}=\cos^{-1}(x-1)$	M1		
	$\cos\left(\frac{y}{2}\right)=x-1$			
	$x=\cos\left(\frac{y}{2}\right)+1$	A1	2	
(ii)	$-\frac{1}{2}\sin\left(\frac{y}{2}\right)$	M1		$k\ \sin\ (...)$
		A1		$\frac{\mathrm{d}x}{\mathrm{d}y}$ correct
	At $y=2,\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right)-\frac{1}{2}\sin 1$	A1	3	Condone AWRT –0.42
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$y=\dfrac{4x}{4x-3}$			
	$\dfrac{dy}{dx}=\dfrac{(4x-3).4-4x(4)}{(4x-3)^2}$	M1		Must use quotient rule Condone one slip
	$=\dfrac{-12}{(4x-3)^2}$	A1	2	$k=-12$
(b)(i)	$y=x\ln(4x-3)$			
	$\dfrac{dy}{dx}=\dfrac{x.4}{4x-3}+\ln(4x-3)$	M1		$\dfrac{f(x)}{4x-3}+g(x)$ ‘f(x)’ may be constant
		m1		$\dfrac{kx}{4x-3}+\ln(4x-3)$
		A1	3	
(ii)	$x=1 \quad y=0$	B1		
	$\dfrac{dy}{dx}=4$	M1		Sub $x=1$ into their $\dfrac{dy}{dx}$
	$\therefore y=4(x-1)$ any correct form	A1	3	CSO Must have full marks in (b)(i)
(c)(i)	$u=4x-3$			
	$du=4dx$	M1		
	$\int \dfrac{4x}{4x-3}dx=\int \dfrac{u+3}{u}\dfrac{du}{4}$	A1		Or $\int \dfrac{4x}{4x-3}dx=\int\left(1+\dfrac{3}{4x-3}\right)dx$
	$=\left(\dfrac{1}{4}\right)\int\left(1+\dfrac{3}{u}\right)(du)$	m1		$=\int\left(1+\dfrac{3}{u}\right)du$ etc
	$=\dfrac{1}{4}(u+3\ln u)$			
	$=\dfrac{1}{4}\left[(4x-3)+3\ln(4x-3)\right](+c)$	A1	4	CSO Condone missing du
(ii)	$\int \ln(4x-3)\,dx$			
	$u=\ln(4x-3) \quad \dfrac{dv}{dx}=1$	M1		In correct direction
	$\dfrac{du}{dx}=\dfrac{4}{4x-3} \quad v=x$			
	$\int = x\ln(4x-3)-\int\dfrac{4x}{4x-3}dx$	A1		
	$=x\ln(4x-3)-\dfrac{1}{4}\left[(4x-3)+3\ln(4x-3)\right]$	m1		$x\ln(4x-3)-$ their (c)(i)
	$(+c)$	A1	4	
	Total		**16**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2009 examination - June series

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Otherwise we require evidence of a correct method for any marks to be awarded.

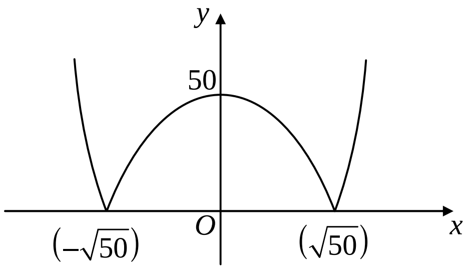
MPC3

Q	Solution	Marks	Total	Comments
1(a)(i)	$f(x)=\frac{\cos x}{2x+1}-\frac{1}{2}$			OE
	$f(0)=\frac{1}{2}$; $f\left(\frac{\pi}{2}\right)=-\frac{1}{2}$	M1		$x=0$ LHS $=1$, $x=\frac{\pi}{2}$ LHS $=0$
	Change of sign $0<\alpha<\frac{\pi}{2}$	A1	2	Either side of $\frac{1}{2}$, $\therefore 0<\alpha<\frac{\pi}{2}$
(ii)	$\frac{\cos x}{2x+1}=\frac{1}{2}$			
	$2\cos x=2x+1$ $2\cos x-1=2x$ } or, $\cos x=x+\frac{1}{2}$			Either line
	$x=\cos x-\frac{1}{2}$	B1	1	AG; or $\cos x-\frac{1}{2}=x$ All correct with no errors
(iii)	$x_1=0$			
	$x_2=0.5$	M1		Attempt at iteration (allow $x_2=-0.5$, $x_3=0.38, 0.4$)
	$x_3=0.378$	A1	2	CAO
(b)(i)	$\frac{dy}{dx}=\frac{(2x+1)(-\sin x)-\cos x\times 2}{(2x+1)^2}$	M1		Attempt at quotient rule: $\frac{\pm(2x+1)\sin x\pm 2\cos x}{(2x+1)^2}$
		A1		Either term correct
		A1	3	All correct ISW
(ii)	$x=0$			
	$\frac{dy}{dx}=-2$	m1		Correctly subst. $x=0$ into their $\frac{dy}{dx}$
	$\therefore$ Gradient of normal $=\frac{1}{2}$	A1	2	CSO
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\mathrm{f}(x) \geqslant 0$	M1		For $\geqslant 0$, $\mathrm{f}(x) > 0$
		A1	2	Correct; allow $y \geqslant 0$, $\mathrm{f} \geqslant 0$
(b)(i)	$y = \sqrt{2x+5}$			
	$x = \sqrt{2y+5}$	M1		$x \Leftrightarrow y$
	$x^2 = 2y+5$	M1		Attempt to isolate, squaring first
	$\mathrm{f}^{-1}(x) = \dfrac{x^2-5}{2}$	A1	3	condone $(y = \ )$
(ii)	$x \geqslant 0$	B1F	1	ft their (a), but **must** be x
2(c)(i)	$\mathrm{h}(x) = \mathrm{fg}(x)$			
	$= \sqrt{2\left(\dfrac{1}{4x+1}\right) + 5}$	B1	1	
(ii)	$\sqrt{2\left(\dfrac{1}{4x+1}\right)+5} = 3$			
	$2\left(\dfrac{1}{4x+1}\right)+5 = 9$	M1		one correct step from (c)(i), squaring
	$\dfrac{1}{4x+1} = 2$ $4x+1 = \dfrac{1}{2}$ } either or $16x+4=2$	A1		
	$x = -\dfrac{1}{8}$ or equiv	A1	3	CSO
	Total		**10**	
3(a)	$\tan^{-1}\left(-\dfrac{1}{3}\right) = -0.32$	M1		Sight of $\pm\, 0.32$ or 18.43
		A1		a correct answer AWRT
	$x = 2.82,\ 5.96$	A1	3	–1 for any extra in range, ignore extra answers not in range. [SC $161.57, 341.57$ AWRT M1A1 (max 2/3)]
(b)	$3\left(\tan^2 x + 1\right) = 5\tan x + 5$			
	$3\tan^2 x - 5\tan x - 2 = 0$	B1	1	AG
3(c)	$(3\tan x + 1)(\tan x - 2) = 0$	M1		Attempt at factorisation/formula
	$\tan x = 2, -\dfrac{1}{3}$	A1		
	$x = 1.11,\ 4.25,\ 2.82,\ 5.96$ AWRT	B1		3 correct [SC $x = 1.11, 4.25$ + their two answers from (a)]
		B1	4	4 correct, no extras in range [SC $161.57, 341.57, 63.43, 243.43$ AWRT B1 (max 3/4)]
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)		M1		Modulus graph, 3 section, condone shape inside + outside $\pm\sqrt{50}$
		A1		Cusps + curvature outside $\pm\sqrt{50}$
		A1	3	Value of y and shape inside $\left(\pm\sqrt{50}\right)$
(b)	$\lvert 50-x^2 \rvert = 14$			
	$50-x^2=14 \quad x^2=36$ $50-x^2=-14 \quad x^2=64$	M1		Either
	$x = \pm 6, \pm 8$	A1		2 correct, from correct working
		A1	3	All 4 correct, from correct working
(c)	$-6 < x < 6$	B1		
	$x > 8,\ x < -8$	B1	2	
(d)	Reflect in x-axis	M1,A1		or $\left\{\text{Reflect in } y=a;\ \text{Translate} \begin{bmatrix}0\\50-2a\end{bmatrix}\right\}$
	Translate $\begin{bmatrix}0\\50\end{bmatrix}$	E1, B1	4	or $\left\{\text{Translate} \begin{bmatrix}0\\-50\end{bmatrix};\ \text{Reflect in } x-\text{axis}\right\}$ or $\left\{\text{Translate} \begin{bmatrix}0\\2a-50\end{bmatrix};\ \text{Reflect in } y=a\right\}$
	Reflect in y = 25 scores 4/4			
	Total		**12**	
5(a)	$2\ln x = 5$			
	$\ln x = \frac{5}{2} \quad x = e^{\frac{5}{2}}$	B1	1	
(b)	$2\ln x + \frac{15}{\ln x} = 11$			
	$2(\ln x)^2 - 11\ln x + 15 = 0$	M1		Forming quadratic equation in $\ln x$, condone poor notation
	$(2\ln x - 5)(\ln x - 3) = 0$	m1		Attempt at factorisation/formula
	$\ln x = \frac{5}{2}, 3$ condone $2\ln x = 5$	A1		
	$x = e^{\frac{5}{2}},\ e^3$	A1,A1	5	[SC for substituting $x = e^{\frac{5}{2}}$ or equivalent into equation and verifying B1 $\left(\frac{1}{5}\right)$]
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$V = \pi\int x^2\,dy$	B1		PI
	$V = \frac{(\pi)}{4}\int (100 - y^2)\,dy$	M1		$k\int(100 - y^2)\,dy$ may be recovered Allow $\int(\text{their } x)^2 dy$, expanded
	$= \frac{(\pi)}{4}\left[100y - \frac{y^3}{3}\right]_{(0)}^{(10)}$	A1		
	$= \frac{(\pi)}{4}\left[\frac{2000}{3}\right]$	m1		For F(10) − F(0)
	$= \frac{500\pi}{3}$	A1	5	OE CSO
				SC: if rotated about x-axis $V = \pi\left[100x - \frac{4x^3}{3}\right]_0^5$ M1 $= \frac{1000}{3}\pi$ A1 max 2/5
(b)	x y 0.5 9.95(0) 1.5 9.539 2.5 8.66(0) 3.5 7.141 4.5 4.359 or better	B1 M1 A1		Correct x 4 + correct y to 2sf All y correct
	$A = 1 \times \sum y = 39.6$	A1	4	(39.6 scores $4/4$)
6(c)(i)	$\frac{dy}{dx} = \frac{1}{2}(100 - 4x^2)^{-\frac{1}{2}}(-8x)$	M1		Chain rule $(\ )^{-\frac{1}{2}} \times f(x)$; allow f($x$) = k $f(x) = \frac{1}{2}(-8x) = -4x$
	$x = 3 \Rightarrow \frac{dy}{dx} = -12(100 - 36)^{-\frac{1}{2}}$	A1		
	$= -\frac{3}{2}$ or equivalent	A1	3	CSO
(ii)	$y - 8 = -\frac{3}{2}(x - 3)$	M1		$y - 8 = \left(\text{their } \frac{dy}{dx}\right)(x-3)$ or $y = \left(\text{their } \frac{dy}{dx}\right)x + c$ **and** subst. (3,8) to find c
	$(2y - 16 = -3x + 9)$ $2y + 3x = 25$	A1	2	AG; all correct with no slips, full marks in part (i)

MPC3

Q	Solution	Marks	Total	Comments
6(d)	$x = 0 \quad y = \frac{25}{2}$ or equivalent	B1		
	$y = 0 \quad x = \frac{25}{3}$	B1		OE
	Area of $\Delta = \frac{1}{2}\times\frac{25}{2}\times\frac{25}{3}$	M1		for $\frac{1}{2}(\text{their } y)\times(\text{their } x)$ or $\frac{1}{2}ab\sin C$
	Area = Area $\Delta -$ (b)	m1		PI $\Delta >$ (b)
	Required area = 12.5 AWRT	A1	5	Condone 12.4 AWRT
(d)	**Alternative**			
	Area $\Delta = \int_0^{\frac{25}{3}} \frac{1}{2}(25-3x)(\mathrm{d}x)$	(B1) (B1)		
	$= \frac{1}{2}\left[25x - \frac{3x^2}{2}\right]_0^{\frac{25}{3}}$ $\frac{1}{2}\left[\frac{625}{3} - \frac{625}{6}\right]$	(M1)		For integration **and** $\mathrm{f}(\frac{25}{3}) - \mathrm{f}(0)$
	$= \frac{625}{12}$			
	Total		**19**	
7(a)	$\int (t-1)\ln t \ \mathrm{d}t$			
	$u = \ln t \quad \frac{\mathrm{d}v}{\mathrm{d}t} = t-1$	M1		Differentiate + integrate, correct direction
	$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{t} \quad v = \frac{t^2}{2} - t$	A1		All correct
	$\int = \left(\frac{t^2}{2} - t\right)\ln t - \int\left(\frac{t^2}{2} - t\right)\times\frac{1}{t}(\mathrm{d}t)$			
	$= \left(\frac{t^2}{2} - t\right)\ln t - \int\left(\frac{t}{2} - 1\right)(\mathrm{d}t)$	A1		Condone missing brackets
	$= \left(\frac{t^2}{2} - t\right)\ln t - \frac{t^2}{4} + t(+c)$	A1	4	CAO

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	**Alternative**			
	$\int (t-1)\ln t$	(M1)		$u = \ln t \quad v' = (t-1)$
		(A1)		$u' = \frac{1}{t} \quad v = \frac{(t-1)^2}{2}$
	$\int = \frac{(t-1)^2}{2}\ln t - \int \frac{(t-1)^2}{t} \frac{1}{t}\,dt$			
	$\frac{(t-1)^2}{2}\ln t - \frac{1}{2}\int \frac{t^2-2t+1}{t}\,dt$			
	$\frac{(t-1)^2}{2}\ln t - \frac{1}{2}\int t-2+\frac{1}{t}\,dt$	(A1)		
	$\frac{(t-1)^2}{2}\ln t - \frac{1}{2}\left[\frac{t^2}{2}-2t+\ln t\right]$	(A1)		
	$=\frac{t^2}{2}\ln t - t\ln t + \frac{1}{2}\ln t - \frac{t^2}{4} + t - \frac{1}{2}\ln t$			
	$=\left(\frac{t^2}{2}-t\right)\ln t - \frac{1}{4}t^2 + t + c$		(4)	
(b)	$t = 2x+1$			
	$dt = 2\,dx$ (RHS)	M1		$\frac{dt}{dx} = 2$ (LHS)
	$2x = t-1$,	m1		OE
	$\int = \int 2(t-1)\ln t \frac{dt}{2}$	A1	3	AG
(c)	$[x]_0^1 = [t]_1^3$	M1		Limit becoming 3
	$\int = \left[\left(\frac{t^2}{2}-t\right)\ln t - \frac{t^2}{4}+t\right]_1^3$			
	$=\left[\left(\frac{9}{2}-3\right)\ln 3 - \frac{9}{4}+3\right]-\left[0-\frac{1}{4}+1\right]$	m1		Correctly sub. 1,3 into their (a)
	$=\frac{3}{2}\ln 3$	A1	3	CSO
	or			
	$\int = \left[\left(\frac{(2x+1)^2}{2}-(2x+1)\right)\ln(2x+1) - \frac{(2x+1)^2}{4}+(2x+1)\right]_0^1$	(M1)		Condone 1 slip
	$=\left(\left(\frac{9}{2}-3\right)\ln 3 - \frac{9}{4}+3\right)-\left(0-\frac{1}{4}+1\right)$	(m1)		Correctly sub. 0,1
	$=\frac{3}{2}\ln 3$	(A1)	(3)	CSO
	Total		**10**	
	TOTAL		**75**	

Version 1.0 0110

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

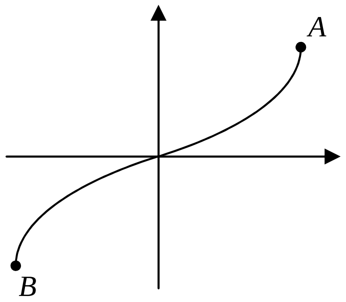
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

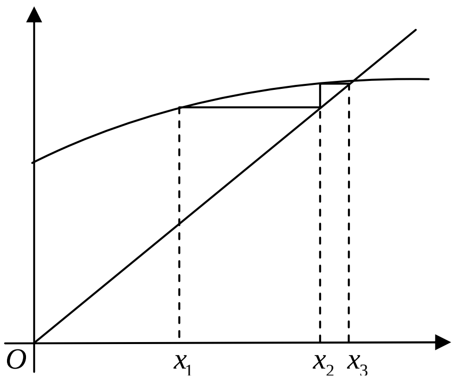
MPC3

Q	Solution	Marks	Total	Comments
1(a)	$y' = e^{-4x}(2x+2) \quad -4e^{-4x}(x^2+2x-2)$	M1		$y' = Ae^{-4x}(ax+b) \pm Be^{-4x}(x^2+2x-2)$ where A and B are non-zero constants
		A1		All correct
	$= e^{-4x}(2x+2-4x^2-8x+8)$			**or** $-4x^2e^{-4x} - 6xe^{-4x} + 10e^{-4x}$
	$= 2e^{-4x}(5-3x-2x^2)$	A1	3	AG; all correct with no errors, 2nd line (OE) must be seen Condone incorrect order on final line
	or $y = x^2 e^{-4x} + 2xe^{-4x} - 2e^{-4x}$			
	$y' = -4x^2e^{-4x} + 2xe^{-4x} + 2x.-4e^{-4x}$ $+2e^{-4x} + 8e^{-4x}$	(M1) (A1)		$Ax^2e^{-4x} + Bxe^{-4x} + Cxe^{-4x} + De^{-4x} + Ee^{-4x}$ All correct
	$= -4x^2e^{-4x} - 6xe^{-4x} + 10e^{-4x}$ $= 2e^{-4x}(5-3x-2x^2)$	(A1)		AG; all correct with no errors, 3rd line (OE) must be seen
(b)	$-(2x+5)(x-1)(=0)$	M1		OE Attempt at factorisation $(\pm 2x \pm 5)(\pm x \pm 1)$ or formula with at most one error
	$x = \frac{-5}{2}, 1$	A1		Both correct and no errors
				SC $x = 1$ only scores M1A0
	$x = 1,\ y = e^{-4}$	m1		For $y = ae^b$ attempted
		A1F		Either correct, follow through only from incorrect sign for x
	$x = -\frac{5}{2},\ y = e^{10}\left(-\frac{3}{4}\right)$	A1	5	CSO 2 solutions only
				Note: withhold final mark for extra solutions Note: approximate values only for y can score m1 only
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)		B1		correct shape passing through origin and stopping at *A* and *B*
	$A\left(1, \frac{\pi}{2}\right)$	B1		
	$B\left(-1, -\frac{\pi}{2}\right)$	B1	3	SC $A(1, 90)$ and $B(-1, -90)$ scores B1
(ii)	line intersecting their curve (positive gradient, positive *y* intercept)	M1		
	Correct statement	A1	2	one solution only, stated or indicated on sketch - must be in the first quadrant (ie curve intersects line once) Must have scored B1 for graph in (a)(i)
(b)	$\left.\begin{array}{ll}\text{LHS }(0.5)=0.5 & \text{RHS}(0.5)=1.1\\ \text{LHS}(1)=1.6 & \text{RHS}(1)=1.3\end{array}\right\}$	M1		
	At 0.5 LHS < RHS , At 1 LHS > RHS $\therefore\ 0.5 < \alpha < 1$	A1	2	CSO
	or			
	$f(x)=\sin^{-1}(x)-\frac{1}{4}x-1$			$f(x)$ must be defined
	$\left.\begin{array}{l}f(0.5)=-0.6\\ f(1)=0.3\end{array}\right\}$ AWRT	(M1)		Allow $f(0.5)<0$ $f(1)>0$
	Change of sign $\Rightarrow 0.5<\alpha<1$	(A1)		
	or			
	$f(x)=\sin\left(\frac{1}{4}x+1\right)-x$			$f(x)$ must be defined
	$\left.\begin{array}{l}f(0.5)=0.4\\ f(1)=-0.1\end{array}\right\}$ Attempt	(M1)		
	Change of sign $\Rightarrow 0.5<\alpha<1$	(A1)		
	or			
	$f(x)=4\sin^{-1}x-x-4$			$f(x)$ must be defined
	$\left.\begin{array}{l}f(0.5)=-2.4\\ f(1)=1.3\end{array}\right\}$ attempt	(M1)		
	Change of sign $\Rightarrow 0.5<\alpha<1$	(A1)		

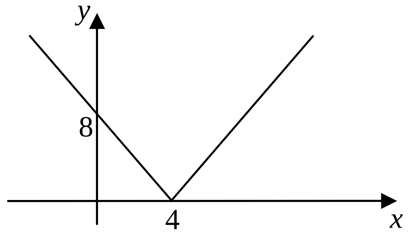
MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(c)(i)	$x_2 = 0.902$	M1		Sight of AWRT 0.902 or AWRT 0.941
	$x_3 = 0.941$	A1	2	These values only
(ii)		M1		Staircase, (vertical line) from x_1 to curve, horizontal to line, vertical to curve
		A1	2	x_2 , x_3 approx correct position on x-axis
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\sin x = \frac{1}{3}$, or sight of $\pm 0.34, \pm 0.11\pi$ or ± 19.47 (or better)	M1		
	$x = 0.34,\ 2.8(0)$ AWRT	A1	2	Penalise if incorrect answers in range; ignore answers outside range
(b)	$\text{cosec}^2 x - 1 = 11 - \text{cosec}\, x$	M1		Correct use of $\cot^2 x = \text{cosec}^2 x - 1$
	$\text{cosec}^2 x + \text{cosec}\, x - 12 (= 0)$	A1		
	$(\text{cosec}\, x + 4)(\text{cosec}\, x - 3)(= 0)$	m1		Attempt at Factors Gives cosec x or – 12 when expanded Formula one error condoned
	$\text{cosec}\, x = -4,\ 3$ $\sin x = -\frac{1}{4}, \frac{1}{3}$	A1		Either Line
	$\sin x = -\frac{1}{4}$ $\Rightarrow x = 3.39,\ 6.03$ AWRT	B1F		3 correct or their two answers from (a) and 3.39, 6.03
	$0.34,\ 2.8(0)$ AWRT	B1	6	4 correct and no extras in range ignore answers outside range SC 19.47, 160.53, 194.48, 345.52 B1
	Alternative $\frac{\cos^2 x}{\sin^2 x} = 11 - \frac{1}{\sin x}$ $\cos^2 x = 11\sin^2 x - \sin x$	(M1)		Correct use of trig ratios and multiplying by $\sin^2 x$
	$1 - \sin^2 x = 11\sin^2 x - \sin x$ $0 = 12\sin^2 x - \sin x - 1$	(A1)		
	$0 = (4\sin x + 1)(3\sin x - 1)$	(m1)		Attempt at factors as above
	$\sin x = -\frac{1}{4}, \frac{1}{3}$	(A1)		
		(B1F) (B1)		As above
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)		M1		Modulus graph V shape in 1st quad going into 2nd quad, touching x-axis. Must cross y-axis Condone not ruled
		A1	2	4 and 8 labelled
(b)	$x = 2$	B1		One correct answer
	$x = 6$	B1	2	Second correct answer and no extras Condone answers shown on the graph, if clearly indicated
(c)	$x > 6$	B1		One correct answer
	$x < 2$	B1	2	Second correct answer and no extras and no further incorrect statement eg $6 < x < 2$ or $2 < x > 6$ SC $x \geq 6$, $x \leq 2$ scores B1
	Total		**6**	
5(a)	x / y: 1.5 / 1.98100 4.5 / 3.22883 7.5 / 4.11496 10.5 / 4.74710	B1		x values correct PI
		M1		3+ y values correct to 2sf or better or exact values
		A1		1.981, 3.228 / 9, 4.114 / 5, 4.747 for y (or better)
	$\int = 3 \times \sum y$ $= 42.2$	A1	4	(Note: 42.2 with evidence of mid-ordinate rule with four strips scores 4/4)
(b)(i)	$y = \ln\left(x^2 + 5\right)$ $e^y = x^2 + 5$			OE
	$x^2 = e^y - 5$	B1	1	AG Must see middle line, and no errors
(ii)	$(\pi)\int\left(e^y - 5\right)(dy)$	M1		Condone omission of brackets around f (y) throughout
	$= (\pi)\left[e^y - 5y\right]_{(5)}^{(10)}$	A1		
	$= (\pi)\left[\left(e^{10} - 50\right) - \left(e^5 - 25\right)\right]$	m1		$F(10) - F(5)$
	$V = \pi\left[e^{10} - e^5 - 25\right]$	A1	4	CSO including correct notation – must see **d**y ISW if evaluated
(c)	$(y =) \ln\left[\left(\frac{x}{4}\right)^2 + 5\right] + 3$	M1		$\frac{x}{4}$ seen, condone $\ln \frac{x^2}{4} +$ ....
		B1		… + 3
		A1	3	CSO mark final answer (no ISW)
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\text{f}(x) > -3$	M1		'>-3', '$x>-3$' or '$\text{f}(x) \geq -3$'
		A1	2	Allow $y > -3$
(b)(i)	$y = \text{e}^{2x} - 3$			
	$y + 3 = \text{e}^{2x}$			
	$\ln(y+3) = 2x$	M1		swap x and y
		M1		attempt to isolate: $\ln(y \pm A) = Bx$ or reverse
	$\left(\text{f}^{-1}(x)\right) = \frac{1}{2}\ln(x+3)$	A1	3	OE with no further incorrect working Condone $y = \ldots\ldots$
	Alternative $x \rightarrow \times 2 \rightarrow \text{e} \rightarrow -3$ $\div 2 \leftarrow \ln \leftarrow +3 \leftarrow x$ (M1) (M1)			
	$y = \dfrac{\ln(x+3)}{2}$	(A1)		
(ii)	$x + 3 = 1$	M1		for putting their $\text{p}(x) = 1$ from $k\ln(\text{p}(x))$ in their part (b)(i)
	$x = -2$	A1	2	CSO SC: B2 x = -2 with no working, if full marks gained in part (b)(i)
(c)(i)	$(\text{gf}(x) =) \dfrac{1}{3(\text{e}^{2x}-3)+4}$ $(=) \dfrac{1}{3\text{e}^{2x}-5}$ } either OE	B1	1	substituting f into g ISW
(ii)	$\dfrac{1}{3\text{e}^{2x}-5} = 1$			
	$1 = 3\text{e}^{2x} - 5$ OE	M1		Correct removal of their fraction
	$\text{e}^{2x} = 2$			
	$2x = \ln 2$	m1		Correct use of logs leading to $kx = \ln\dfrac{a}{b}$
	$x = \frac{1}{2}\ln 2$ OE	A1	3	CSO No ISW except for numerical evaluation
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left(\frac{dy}{dx}=\right)\frac{\cos 4x.4\cos 4x-\sin 4x.-4\sin 4x}{\cos^2 4x}$	M1		$\frac{\pm A\cos^2 4x \pm B\sin^2 4x}{\cos^2 4x}$
	$=\frac{4\cos^2 4x+4\sin^2 4x}{\cos^2 4x}$ or better	A1		Both terms correct
	$=4\left(1+\tan^2 4x\right)$ CSO	A1	3	All correct
	or			
	$\left(\frac{dy}{dx}=\right)\frac{\cos 4x.4\cos 4x-\sin 4x.-4\sin 4x}{\cos^2 4x}$	(M1)		$\frac{\pm A\cos^2 4x \pm B\sin^2 4x}{\cos^2 4x}$
	$=\frac{4\cos 4x\cos 4x}{\cos 4x\cos 4x}+\frac{4\sin 4x\sin 4x}{\cos 4x\cos 4x}$ or better	(A1)		
	$=4\left(1+\tan^2 4x\right)$ CSO	(A1)		All correct
(b)	$\frac{d^2y}{dx^2}=4\times 2\tan 4x\times \ldots\ldots$	M1		$A\tan 4x\times f(4x)$
	$4\sec^2 4x$	m1		$f(4x) = B\sec^2 4x$
	$=32\tan 4x\sec^2 4x$	A1F		ft $8\times$ their p from part (a)
	$=32\tan 4x\left(1+\tan^2 4x\right)$	m1		Previous two method marks must have been earned
	$=32y\left(1+y^2\right)$	A1	5	CSO
	Alternative Solutions			
	$y' = 4+4\tan^2 4x = 4+4\frac{\sin^2 4x}{\cos^2 4x}$			
	$y''=4\times\left[\frac{\left[\cos^2 4x\,2\sin 4x4\cos 4x+\sin^2 4x\,2\cos 4x\,4\sin 4x\right]}{\cos^4 4x}\right]$	(M1)		$\frac{A\cos^3 4x\pm B\sin^3 4x}{\cos^4 4x}$ where A and B are constants or trig functions.
		(m1)		Where A is $m\sin 4x$ and B is $n\cos 4x$
	$=\frac{4\times 8\sin 4x\cos 4x\left[\cos^2 4x+\sin^2 4x\right]}{\cos^4 4x}$	(A1F)		ft $8\times$ their p from part (a)
	$=32\tan 4x\sec^2 4x$	(m1)		$k\tan 4x\sec^2 4x$
	$=32y\left(1+y^2\right)$	(A1)		CSO
	or			
	$\frac{dy}{dx}=4\sec^2 4x$			
	$\frac{d^2y}{dx^2}=4\times 2\sec 4x.4\sec 4x\tan 4x$	(M1) (m1)		$A\sec 4x\times f(4x)$ $f(4x) = B\sec 4x\tan 4x$
	$=32\sec^2 4x\tan 4x$	(A1F)		ft $8\times$ their p from part (a)
	$=32\left(1+\tan^2 4x\right)\tan 4x$	(m1)		Previous two method marks must have been earned
	$=32y\left(1+y^2\right)$	(A1)		CSO

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(b) or	$\frac{dy}{dx} = 4(1+\tan^2 4x)$			
	$u = \tan 4x \quad \frac{dy}{dx} = 4 + 4u^2$			
	$\frac{d^2y}{dx^2} = (8)u\frac{du}{dx}$	(M1)		
	$\frac{du}{dx} = 4 + 4\tan^2 4x = 4 + 4u^2$	(m1)		
	$\frac{d^2y}{dx^2} = 8u(4+4u^2)$	(A1)		
	$= 32u(1+u^2)$	(m1)		
	$= 32y(1+y^2)$	(A1)		
	Total		**8**	
8(a)	$\int x\sin(2x-1)\,dx$			
	$u = x \quad \frac{dv}{dx} = \sin(2x-1)$	M1		$\int \sin f(x),\ \frac{d}{dx}(x)$ attempted
	$\frac{du}{dx} = 1 \quad v = -\frac{1}{2}\cos(2x-1)$	A1		All correct – condone omission of brackets
	$(\int =) -\frac{x}{2}\cos(2x-1)$ $-\int -\frac{1}{2}\cos(2x-1)\ (dx)$	m1		correct substitution of their terms into parts
	$= -\frac{x}{2}\cos(2x-1) + \frac{1}{2}\int\cos(2x-1)\ (dx)$	A1		All correct – condone omission of brackets
	$= -\frac{x}{2}\cos(2x-1) + \frac{1}{4}\sin(2x-1) + c$	A1	5	CSO condone missing + c and dx Condone missing brackets around $2x - 1$ if recovered in final line ISW
(b)	$u = 2x - 1$ '$du = 2\,dx$'	M1		OE
	$\int \frac{x^2}{2x-1}\,dx = \int \frac{(u+1)^2}{4u}\frac{du}{2}$	m1 A1		All in terms of u All correct PI from later working
	$= \left(\frac{1}{8}\right)\int \frac{u^2+2u+1}{u}\,du$			
	$= \left(\frac{1}{8}\right)\int u + 2 + \frac{1}{u}\,du$	A1		
	$= \left(\frac{1}{8}\right)\left[\frac{u^2}{2} + 2u + \ln u\right]$	B1		or $\left(\frac{1}{8}\right)\left[\frac{(u+2)^2}{2} + \ln u\right]$
	$= \frac{1}{8}\left[\frac{(2x-1)^2}{2} + 2(2x-1) + \ln(2x-1)\right] + c$	A1	6	or $= \frac{1}{8}\left[\frac{(2x+1)^2}{2} + \ln(2x-1)\right] + c$ CSO condone missing + c only ISW
	Total		**11**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education
June 2010

Mathematics MPC3

Pure Core 3

Mark Scheme

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Registered address: AQA, Devas Street, Manchester M15 6EX

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

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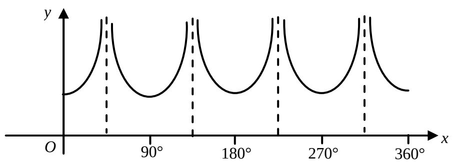
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$f(x) = 3^x - 10 + x^3$ (or reverse) $f(1) = -6$ $f(2) = 7$	M1		Attempt to evaluate f(1) and f(2)
	Change of sign $\therefore 1 < \alpha < 2$	A1	2	All working must be correct plus statement
	OR			
	LHS $(1) = 3$ RHS $(1) = 9$ LHS $(2) = 9$ RHS $(2) = 2$	(M1)		Must be these values
	At 1 LHS < RHS, At 2 LHS > RHS $\therefore 1 < \alpha < 2$	(A1)		
(b)(i)	$3^x = 10 - x^3$ $x^3 = 10 - 3^x$ $x = \sqrt[3]{10 - 3^x}$	B1	1	This line must be seen AG
(ii)	$(x_1 = 1)$ $x_2 = 1.913$ $x_3 = 1.221$	M1 A1	2	Sight of AWRT 1.9 or AWRT 1.2 Both values correct
	Total		**5**	

MPC3

Q	Solution	Marks	Total	Comments
2(a)(i)	$(y=)\ 1$	B1	1	Condone 1 marked at A, $A = 1$ etc but not $\dfrac{1}{\cos 0}$, sec 0
(ii)		M1		Modulus graph $y>0$
		A1		$3+2\times\frac{1}{2}$ sections roughly as shown, condone sections touching, variable minimum heights
		A1	3	Correct graph with correct behaviour at 4 asymptotes but need not show broken lines; and roughly same minima
(b)	$\cos x=\frac{1}{2}$ or $\cos^{-1}\frac{1}{2}$ seen	M1		or sight of $\pm 60^\circ$ or $\pm\frac{\pi}{3}$, ± 1.05 (AWRT)
	$x = 60^\circ, 300^\circ$	A1	2	Condone extra values outside $0° < x < 360°$, but no extras in interval
(c)	$\sec(2x-10^\circ)=2$, $\sec(2x-10^\circ)=-2$ $\cos(2x-10^\circ)=\frac{1}{2}$ or $\cos(2x-10^\circ)=-\frac{1}{2}$	M1		Either of these, PI by further working
	$2x-10^\circ=60^\circ, 300^\circ$ or $2x-10^\circ=120^\circ,\ 240^\circ$ (ignore values outside $0° < x < 360°$)	A1		Both correct values from one equation or 2 correct values and no wrong values from both equations, but must have "$2x-10^\circ=$" PI by $2x=70^\circ, 130^\circ, 250^\circ, 310^\circ$
	$x=35^\circ, 65^\circ, 125^\circ, 155^\circ$	B1		3 correct (and not more than 1 extra value in $0° < x < 180°$)
		B1	4	All 4 correct (and no extras in interval)
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$y = \ln(5x-2)$			
		M1		$\frac{k}{5x-2}$
	$\left(\frac{dy}{dx}=\right)\ \frac{5}{5x-2}$	A1	2	No ISW, eg $\frac{5}{5x-2}=\frac{1}{x-2}$ (M1A0)
(ii)	$y = \sin 2x$			
		M1		$k\ \cos 2x$
	$\left(\frac{dy}{dx}=\right)\ 2\cos 2x$	A1	2	
(b)(i)		M1		
	$f(x) \geqslant \ln 0.5$ or $f(x) \geqslant -\ln 2$	A1	2	
(ii)	$(gf(x)=)\quad \sin[2\ln(5x-2)]$ or $(gf(x)=)\quad \sin\ln(5x-2)^2$	B1	1	Condone $\sin 2\ln(5x-2)$ or $\sin 2(\ln(5x-2))$ but **not** $\sin 2(\ln 5x-2)$ or $\sin 2\ln 5x-2$
(iii)	$gf(x)=0$ $\sin[2\ln(5x-2)]=0$			
	$2\ln(5x-2)=0$	M1		Correct first step from their (b)(ii)
	$5x-2=1$	m1		Their $f(x)=1$ from $k\ln(f(x))=0$
	$x=\frac{3}{5}$	A1	3	Withhold if clear error seen other than omission of brackets
(iv)	$x=\sin 2y$			
	$\sin^{-1}x = 2y\quad (\text{or } \sin^{-1}y = 2x)$	M1		Correct equation involving $\sin^{-1}$
	$(g^{-1}(x)=)\ \frac{1}{2}\sin^{-1}x$	A1	2	
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)				
	x = 0.5, $y = \frac{4}{9} = 0.\dot{4}$	B1		x values correct PI
	x = 0.75, $y = \frac{48}{91} = 0.5275$	B1		At least 5 y values that would be correct to 2sf or better, or exact values. May be seen within working.
	x = 1, $y = \frac{1}{2} = 0.5$			
	x = 1.25, $y = \frac{80}{189} = 0.4233$			
	x = 1.5, $y = \frac{12}{35} = 0.3429$			
	x = 1.75, $y = \frac{112}{407} = 0.2752$			
	x = 2, $y = \frac{2}{9} = 0.\dot{2}$			
	$\left[\left(\frac{4}{9}+\frac{2}{9}\right)+4\left(\frac{48}{91}+\frac{80}{189}+\frac{112}{407}\right)+2\left(\frac{1}{2}+\frac{12}{35}\right)\right]$	M1		Clear attempt to use 'their' y values within Simpson's rule
	$\int = \frac{1}{3}\times 0.25[\ \ ]$			
	$= 0.605$	A1	4	Answer must be 0.605 with no extra sf (Note 0.605 with no evidence of Simpson's rule scores 0/4)
(b)	$\int_0^1 \frac{x^2}{1+x^3}\,dx$			
	$= \frac{1}{3}\ln\left(1+x^3\right)$	M1		$k\ln\left(1+x^3\right)$ condone missing brackets
		A1		Correct. A1 may be recovered for missing brackets if implied later
	$= \frac{1}{3}\ln(1+1)\ \left(-\frac{1}{3}\ln 1\right)$	m1		F(1) (– F(0))
	$= \frac{1}{3}\ln 2$	A1	4	ln 1 must not be left in final answer
	Alternative			
	$u = 1+x^3 \quad du = 3x^2dx$			
	$\int = \int \frac{du}{3u}$	(M1)		$\frac{du}{dx}$ correct and integral of form $k\int\frac{du}{u}$
	$= \frac{1}{3}[\ln u]$	(A1)		
	$= \frac{1}{3}\ln 2\ \left(-\frac{1}{3}\ln 1\right)$	(m1)		Correct substitution of correct u limits or conversion back to x and F(1) (– F(0))
	$= \frac{1}{3}\ln 2$	(A1)		ln 1 must not be left in final answer
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$10\operatorname{cosec}^2 x = 16 - 11\cot x$ $10(1+\cot^2 x) = 16 - 11\cot x$ $10\cot^2 x + 11\cot x - 6 = 0$	B1	1	**AG** Must see evidence of correct identity and no errors.
(b)	Attempt at factors, giving $\pm 10\cot^2 x \pm 6$ when expanded.	M1		Use of formula: condone one error
	$(5\cot x - 2)(2\cot x + 3) \quad (=0)$	A1		Correct factors
	$\left(\cot x = \frac{2}{5}, -\frac{3}{2}\right)$			
	$\tan x = \frac{5}{2}, -\frac{2}{3}$	A1,A1	4	1st A1 must be earned Condone AWRT –0.67 ISW if x values attempted
	Alternative 1 $10\cot^2 x + 11\cot x - 6 = 0$ $10\frac{\cos^2 x}{\sin^2 x} + 11\frac{\cos x}{\sin x} - 6 = 0$ $10\cos^2 x + 11\cos x \sin x - 6\sin^2 x = 0$			
	$(5\cos x - 2\sin x)(2\cos x + 3\sin x) \quad (=0)$	(M1)		Attempt at factors, gives $\pm 10\cos^2 x \pm 6\sin^2 x$ when explained
		(A1)		As above
	$(5\cos x = 2\sin x \qquad 2\cos x = -3\sin x)$			
	$\frac{5}{2} = \tan x \qquad -\frac{2}{3} = \tan x$	(A1), (A1)		1st A1 must be earned Condone AWRT –0.67 ISW if x values attempted
	Alternative 2 $10 + 11\tan x - 6\tan^2 x = 0$			
	$(5 - 2\tan x)(2 + 3\tan x) \quad (=0)$	(M1) (A1)		Attempt at factors gives $\pm 10 \pm 6\tan^2 x$
	$\tan x = \frac{5}{2}, -\frac{2}{3}$	(A1), (A1)		1st A1 must be earned Condone AWRT –0.67 ISW if x values attempted
	Total		**5**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$y=\frac{\ln x}{x}$			
	(when) $y=0$ $x=1$ or $(1, 0)$	B1	1	Both coordinates must be stated, not 1 simply shown on diagram
(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$ $\frac{x\times\frac{1}{x}-\ln x}{x^2}$	M1		Quotient/product rule $\frac{\pm\frac{x}{x}\pm\ln x}{x^2}$
	$=\frac{1-\ln x}{x^2}$ or $x^{-2}-x^{-2}\ln x$	A1		OE must simplify $\frac{x}{x}$
	At B, $\frac{1-\ln x}{x^2}=0$	m1		Putting their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or numerator = 0
	$x=\mathrm{e}$	A1		CSO condone $x=\mathrm{e}^1$
	$y=\frac{1}{\mathrm{e}}$ or e^{-1}	A1	5	CSO must simplify ln e
(c)	Gradient at $x=\mathrm{e}^3$			
	$=\frac{1-\ln \mathrm{e}^3}{(\mathrm{e}^3)^2}$	M1		Substituting $x=\mathrm{e}^3$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ (condone 1 slip) but must have scored M1 in (b)
	$=\frac{-2}{\mathrm{e}^6}$ or $-2\mathrm{e}^{-6}$	A1		PI
	Gradient of normal $=\frac{1}{2}\mathrm{e}^6$	A1	3	CSO simplified to this
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\int x\cos 4x \, dx \quad u = x \quad \frac{dv}{dx} = \cos 4x$	M1		$\int \cos 4x$, $\frac{d}{dx}(x)$ attempted
	$\frac{du}{dx} = 1 \quad v = \frac{\sin 4x}{4}$	A1		All correct
	$\int = x\frac{\sin 4x}{4} - \int \frac{\sin 4x}{4} \, dx$	m1		Correct substitution of their terms into parts formula
	$= \frac{x\sin 4x}{4} + \frac{\cos 4x}{16} \; (+c)$	A1	4	OE with fractions unsimplified
(ii)	$\int x^2 \sin 4x \, dx \quad u = x^2 \quad \frac{dv}{dx} = \sin 4x$; $\frac{du}{dx} = 2x \quad v = -\frac{\cos 4x}{4}$	M1		$\int \sin 4x$, $\frac{d}{dx}(x^2)$ attempted
	$\int = \frac{-x^2\cos 4x}{4} - \int \frac{-2x\cos 4x}{4} \, dx$	A1		
	$= \frac{-x^2\cos 4x}{4} + \frac{1}{2}\int x\cos 4x \, dx$			
	$= \frac{-x^2\cos 4x}{4} + \frac{1}{2}[\;\;]$			
	$\left[\frac{x\sin 4x}{4} + \frac{\cos 4x}{16}\right]$	m1		Clear attempt to replace integral using their answer from part (a)(i)
	$= \frac{-x^2\cos 4x}{4} + \frac{x\sin 4x}{8} + \frac{\cos 4x}{32} \; (+c)$	A1	4	OE with fractions unsimplified
(b)	$V = (\pi)\int_{(0)}^{(0.2)} (64) x^2 \sin 4x \, (dx)$	M1		
	$= (\pi \times 64)\left[\frac{-x^2\cos 4x}{4} + \frac{x\sin 4x}{8} + \frac{\cos 4x}{32}\right]$			Must see evidence of their (a)(ii) result or starting again obtaining 3 terms of the form $\pm Ax^2\cos 4x \pm Bx\sin 4x \pm C\cos 4x$
	$= \pi[2.09529 - 2]$	m1		**AND** F(0.2) – F(0) attempted
	$= 0.299$ AWRT	A1	3	Accept AWRT 0.0953π
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$y = e^x \to e^{2x} - 1$ Stretch (I) scale factor $\frac{1}{2}$ (II)	M1		I + (II or III)
	in x-direction (III)	A1		I + II + III
	Translation	E1		Allow “translate”
	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	B1	4	OE “1 unit down” etc
(b)	$x = 0 \quad y = 6$ or $(0, 6)$	B1	1	Both coordinates must be stated, not simply 6 marked on diagram
(c)(i)	$e^{2x} - 1 = 4e^{-2x} + 2$ $e^{4x} - e^{2x} = 4 + 2e^{2x}$ or $\left(e^{2x}\right)^2 - e^{2x} = 4 + 2e^{2x}$	M1		Multiplying both sides by e^{2x}
	$(e^{2x})^2 - 3e^{2x} - 4 = 0$	A1	2	AG With no errors seen
(ii)	$\left(e^{2x} - 4\right)\left(e^{2x} + 1\right)$	M1		$\left(e^{2x} \pm 4\right)\left(e^{2x} \pm 1\right)$
	$x = \ln 2$ or $\frac{1}{2}\ln 4$	A1		
	Reject $e^{2x} = -1$ OE	A1	3	eg $e^{2x} > 0$, $e^{2x} \neq -1$, impossible etc
(d)	$\int\left(4e^{-2x} + 2\right)\,dx$ (I) $= \left[\frac{4e^{-2x}}{-2} + 2x\right]_0^{\ln 2}$	M1		I or II attempted and e^{-2x} or e^{2x} integrated correctly
	$= \left(\frac{4e^{-2\ln 2}}{-2} + 2\ln 2\right) - \left(\frac{4}{-2} + 0\right)$	m1		F[‘their $\ln 2$’ from (c)(ii)] – F[0]
	$= -\frac{1}{2} + 2\ln 2 + +2 = \frac{3}{2} + 2\ln 2$ $\int\left(e^{2x} - 1\right)\,dx$ (II) $= \left[\frac{e^{2x}}{2} - x\right]_0^{\ln 2}$	A1		Both I and II correctly integrated
	$= \left(\frac{e^{2\ln 2}}{2} - \ln 2\right) - \left(\frac{1}{2} - 0\right)$ $= 2 - \ln 2 - \frac{1}{2} = \frac{3}{2} - \ln 2$			
	$A = \left(\frac{3}{2} + 2\ln 2\right) - \left(\frac{3}{2} - \ln 2\right)$	B1√		Attempt to find difference of ‘their I – their II’
	$= 3\ln 2$ or $\ln 8$ or $\frac{3}{2}\ln 4$ OE	A1	5	CSO must be exact

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(d)	**Alternative**			
	$A = \int \left(4e^{-2x} + 2\right) dx - \int \left(e^{2x} - 1\right) dx$	(B1)		Condone functions reversed
	$= \int_{(0)}^{(\ln 2)} \left(4e^{-2x} - e^{2x} + 3\right) dx$			
	$= \left[\frac{4e^{-2x}}{-2} - \frac{e^{2x}}{2} + 3x\right]_0^{\ln 2}$	(M1) (A1)		e^{2x} or e^{-2x} correctly integrated
	$= \left(-2e^{-2\ln 2} - \frac{1}{2}e^{2\ln 2} + 3\ln 2\right) - \left(-2 - \frac{1}{2}\right)$	(m1)		Correct substitution of their $\ln 2$ from (c)(ii) into their integrated expression
	$= 3\ln 2$ or $\ln 8$ or $\frac{3}{2}\ln 4$ OE	(A1)		CSO must be exact
	Total		**15**	
	TOTAL		**75**	

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General Certificate of Education (A-level)
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Mathematics MPC3

(Specification 6360)

Pure Core 3

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Otherwise we require evidence of a correct method for any marks to be awarded.

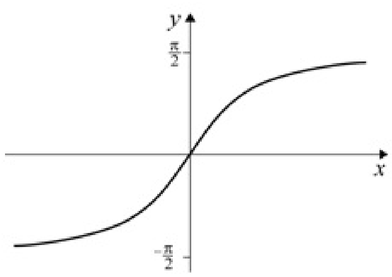
MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = k\left(x^3 - 1\right)^5$	M1		Where k is an integer or function of x
	$= 6 \times 3x^2\left(x^3 - 1\right)^5$ (ISW)	A1	2	
				But note $\frac{\mathrm{d}y}{\mathrm{d}x} = k\left(x^3 - 1\right)^5 + \mathrm{p}x^2$ M0
				Or $\left(u = x^3 - 1\right)$ $\left(y = u^6\right)$
				$\frac{\mathrm{d}y}{\mathrm{d}u} = 6u^5$ and $\frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2$ M1
				$= 6\left(x^3 - 1\right)^5 \times 3x^2$ A1
				Note $\frac{\mathrm{d}y}{\mathrm{d}x} = 6 \times 3x^2\left(x^3 - 1\right)^5 + c$ scores M1 A0 (penalise + c in differential once only in paper)
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm x \times \frac{1}{x} \pm \ln x$	M1		Product rule attempted **and** differential of $\ln x$
	$= 1 + \ln x$ (ISW)	A1	2	
(ii)	$(x = \mathrm{e})$ $y = \mathrm{e}$ PI	B1		Must have replaced ln e by 1 Condone $y = 2.72$ (AWRT)
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln \mathrm{e}$ $(= 2)$	M1		Correct substitution into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ But must have scored M1 in (b)(i)
	$y - \mathrm{e} = 2(x - \mathrm{e})$ or $y = 2x - \mathrm{e}$ OE, ISW	A1	3	Must have replaced ln e by 1
	Total		**7**	

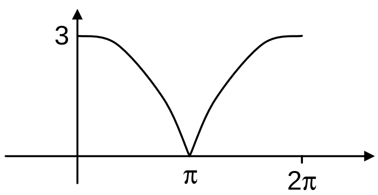
MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$f(x) = (x^2 - 4)\ln(x+2) - 15$ $\left.\begin{matrix} f(3.5) = -0.9 \\ f(3.6) = 0.4 \end{matrix}\right\}$ Attempt at evaluating both f (3.5) and f (3.6)	M1		**Or** reverse $\left.\begin{matrix} f(3.5) = 0.9 \\ f(3.6) = -0.4 \end{matrix}\right\}$ M1 But must see $f(x) = 15 - (x^2 - 4)\ln(x+2)$ before A1 may be earned Condone $\left.\begin{matrix} f(3.5) < 0 \\ f(3.6) > 0 \end{matrix}\right\}$ **Only** if $f(x)$ defined M1 **Or** $\left.\begin{matrix} x = 3.5 \ \ y = 14.1\,(<15) \\ x = 3.6 \ \ y = 15.4\,(>15) \end{matrix}\right\}$ M1
	Change of sign, $\therefore 3.5 < \alpha < 3.6$ OE	A1	2	Either side of 15, $\therefore 3.5 < \alpha < 3.6$ OE A1
(b)	$(x^2 - 4)\ln(x+2) = 15$ $x^2 - 4 = \dfrac{15}{\ln(x+2)}$ $x^2 = 4 + \dfrac{15}{\ln(x+2)}$	M1		Either of these lines correct Condone poor use of brackets for M1 only
	$x = \pm\sqrt{4 + \dfrac{15}{\ln(x+2)}}$ AG	A1	2	Must have **both** middle lines and no errors seen
(c)	$(x_1 = 3\cdot 5)$ $x_2 = 3.578$ CAO	B1		
	$x_3 = 3.568$ CAO	B1	2	Sight of AWRT 3.58 or 3.57 scores B1 B0 Or $\pm$ 3.578 or $\pm$ 3.568 scores B1 B0 $x_1 = 3.578$, $x_2 = 3.568$ scores B1B0
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\frac{\mathrm{d}x}{\mathrm{d}y} = k\sec^2(3y+1)$	M1		Where k is an integer Condone omission of $\frac{\mathrm{d}x}{\mathrm{d}y}$ But $\frac{\mathrm{d}y}{\mathrm{d}x} = k\sec^2(3y+1)$ scores M1 A0
	$= 3\sec^2(3y+1)$ ISW	A1	2	**Alternative methods** $y = \frac{1}{3}(\tan^{-1}x - 1)$ $\frac{\mathrm{d}x}{\mathrm{d}y} = k(1+x^2)$ M1 $= 3(1+\tan^2(3y+1))$ A1 **Or** $x = \frac{\sin(3y+1)}{\cos(3y+1)}$ $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\pm k\cos^2(3y+1) \pm k\sin^2(3y+1)}{\cos^2(3y+1)}$ M1 $= \frac{3}{\cos^2(3y+1)}$ A1
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 3\sec^2\left(3\times -\frac{1}{3} + 1\right)$	M1		Substitution of $y = -\frac{1}{3}$ into their $\frac{\mathrm{d}x}{\mathrm{d}y}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x}$ BUT must have scored M1 in (a)(i)
	$= 3\sec^2 0$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}$ CSO	A1	2	Condone 0.333 or better **Or** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3\sec^2(3y+1)}$ $= \frac{1}{3\sec^2 0}$ $= \frac{1}{3}$ } As above
3(b)		M1		Approx correct shape with no turning points, through (0,0) and only 1 curve
		A1	2	Asymptotic at both $\pm\frac{\pi}{2}$ and both values shown Condone ± 90 (degrees) Condone $y = \tan x$ also drawn but clearly identified, otherwise M0
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$-3 \leqslant f(x) \leqslant 3$	M1 A1	 2	$-3 \leqslant x \leqslant 3,\ -3 < f(x) < 3$ $-3 < f < 3,\ -3 < y < 3$ $-3 \leqslant f < 3,\ -3 < f \leqslant 3$ Allow $-3 \leqslant y \leqslant 3,\ -3 \leqslant f \leqslant 3$
(b)(i)	$y = 3\cos\frac{1}{2}x$ $\frac{y}{3} = \cos\frac{1}{2}x$ $\cos^{-1}\frac{y}{3} = \left(\frac{1}{2}x\right)$ $x = 2\cos^{-1}\frac{y}{3}$ $y = 2\cos^{-1}\frac{x}{3}$ $f^{-1}(x) = 2\cos^{-1}\frac{x}{3}$	 M1 M1 A1	 3	 Or $\cos^{-1}\frac{x}{3} =$ } Either order Swap x and y }
(ii)	$\frac{x}{3} = \cos\frac{1}{2}$ $x = 3\cos\frac{1}{2}$ ISW	M1 A1	 2	If incorrect in (b)(i) **BUT** answer in form $p\cos^{-1}(qx)$ (condone p, q =1) Then $qx = \cos\left(\frac{1}{p}\right)$ M1 **or** $x = f(1)$ M1 $x = 3\cos\frac{1}{2}$ A1
(c)(i)	$gf(x) = \left\|3\cos\frac{1}{2}x\right\|$	B1	1	
(ii)		M1 A1 A1	 3	Modulus graph in 1st quadrant, starting from a +ve y-intercept, at least 2 continuous parts, first descending, then second increasing IGNORE CURVE OUTSIDE RANGE Correct curvature, curves reaching x-axis, condone multiple curves (no turning points at axis) Approximately symmetrical graph with 3, π, 2π indicated (must have scored previous 2 marks) Condone $y = 3\cos\frac{1}{2}x$ also drawn but clearly identified, otherwise M0
(d)	STRETCH + direction s.f. 3, parallel to y-axis s.f. 2, parallel to x-axis	M1 A1 A1	 3	Either in x-direction or y-direction Either order
	Total		**14**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\int \frac{1}{3+2x}\,dx$			
	$= k\ln(3+2x)$	M1		Where k is a rational number
	$= \frac{1}{2}\ln(3+2x)+c$	A1	2	
				Or if substitution $u=3+2x$, $du=2dx$ $\int = \int \frac{1}{u}\frac{du}{2} = k\ln u$ M1 $= \frac{1}{2}\ln(3+2x)+c$ A1
(b)	$u=x \quad dv=\sin\frac{x}{2}$	M1		$\int \sin\frac{x}{2}(dx) = k\cos\frac{x}{2}$, $\frac{d}{dx}(x)=1$ where k is a constant
	$du=1 \quad v=-2\cos\frac{x}{2}$	A1		All correct
	$\int = -2x\cos\frac{x}{2} - \int -2\cos\frac{x}{2}(dx)$	m1		**Correct** substitution of their terms into parts formula (watch signs carefully)
	$= -2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + c$	A1	4	CAO
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)		B1		Using 4 correct *x*-values, PI
	x y			
	0.05 $\cos\sqrt{1.15}$ = 0.4780 0.15 $\cos\sqrt{1.45}$ = 0.3585 0.25 $\cos\sqrt{1.75}$ = 0.2454 0.35 $\cos\sqrt{2.05}$ = 0.1386	M1		At least 3 correct *y*-values, (condone unsimplified correct expressions), Or correct values rounded to 2 s.f. or truncated to 2 s.f.
	$0.1\times\Sigma y$	m1		Used and must be working in radians
	$= 0.122$ CAO	A1	4	Must be 3 s.f.
(b)	$\dfrac{\mathrm{d}u}{\mathrm{d}x}=3$	M1		$\mathrm{d}u=3\mathrm{d}x$ OE
	$\int=\int\left(\dfrac{u\pm1}{3}\right)\sqrt{u}\times k\,\mathrm{d}u$	m1		All in terms of *u*, with $k = 3$ or $\dfrac{1}{3}$ Condone omission of d*u*
	$=\left(\dfrac{1}{9}\right)\int u^{\frac{3}{2}}\pm u^{\frac{1}{2}}\,(\mathrm{d}u)$	m1		$p\int u^{\frac{3}{2}}\pm u^{\frac{1}{2}}\,(\mathrm{d}u)$ (must have scored first 2 marks)
	$=\dfrac{1}{9}\left[\dfrac{u^{\frac{5}{2}}}{\frac{5}{2}}-\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]$	A1		OE
	$=\left(\dfrac{1}{9}\right)\left[\left(\dfrac{2}{5}\times4^{\frac{5}{2}}-\dfrac{2}{3}\times4^{\frac{3}{2}}\right)-\left(\dfrac{2}{5}-\dfrac{2}{3}\right)\right]$	m1		Must have earned all previous method marks and then correct substitution, into their integral, of 1, 4 for *u* or 0, 1 for *x* **and** subtracting
	$=\dfrac{116}{135}$ ISW	A1	6	Or equivalent fraction
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\cos x = -0.2$	M1		Or $\tan x = (\pm)\sqrt{24}$
	$x = 1.77,\ 4.51$ AWRT	A1		One correct value
		A1	3	Second correct value and no extra values in interval 0 to 6.28… Ignore answers outside interval SC $x = 1.8,\ 4.5$ with or without working M1 A1 A0 SC (using degrees) 101.54, 281.54 M1 A1 A0 101.5, 281.5 M1 A0 A0 SC No working shown 2 correct answers 3/3 1 correct answer 2/3
(b)	LHS $= \dfrac{\operatorname{cosec} x(1-\operatorname{cosec} x) - \operatorname{cosec} x(1+\operatorname{cosec} x)}{(1+\operatorname{cosec} x)(1-\operatorname{cosec} x)}$	M1		Correctly combining fractions but condone poor use, or omission, of brackets
	$= \dfrac{\operatorname{cosec} x - \operatorname{cosec}^2 x - \operatorname{cosec} x - \operatorname{cosec}^2 x}{1-\operatorname{cosec}^2 x}$	A1		Allow recovery from incorrect brackets
	$= \dfrac{-2\operatorname{cosec}^2 x}{-\cot^2 x}$ or $\dfrac{-2(1+\cot^2 x)}{-\cot^2 x}$	m1		Correct use of relevant trig identity eg $\operatorname{cosec}^2 x = 1 + \cot^2 x$
	$2\sec^2 x = 50$ $\sec^2 x = 25$ AG	A1	4	All correct with no errors seen INCLUDING correct brackets on 1st line
	Or $\dfrac{\operatorname{cosec} x}{1+\operatorname{cosec} x} - \dfrac{\operatorname{cosec} x}{1-\operatorname{cosec} x} = 50$			
	$\operatorname{cosec} x(1-\operatorname{cosec} x) - \operatorname{cosec} x(1+\operatorname{cosec} x)$ $= 50(1+\operatorname{cosec} x)(1-\operatorname{cosec} x)$	(M1)		Correctly eliminating fractions but condone poor use, or omission, of brackets
	$\operatorname{cosec} x - \operatorname{cosec}^2 x - \operatorname{cosec} x - \operatorname{cosec}^2 x$ $= 50(1-\operatorname{cosec}^2 x)$	(A1)		Allow recovery from incorrect brackets
	$48\operatorname{cosec}^2 x = 50$ $\sin^2 x = \dfrac{24}{25} \Rightarrow \cos^2 x = \dfrac{1}{25}$	(m1)		Correct use of relevant trig identity eg $\sin^2 x = 1 - \cos^2 x$
	$\sec^2 x = 25$ AG	(A1)		All correct with no errors seen INCLUDING correct brackets on 1st line

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(c)	$\sec x = \pm 5$	M1		Or $\cos x = \pm\ 0.2$ Or $\tan x = \pm\sqrt{24}$
	$x = 1.77,\ 4.51,\ 1.37,\ 4.91$ (AWRT)	A1		3 correct
		A1	3	4 correct and no other answers in interval Ignore answers outside interval SC 1.8, 4.5, 1.4, 4.9 With or without working M1 A1 SC their 2 answers from (a) +1.37, 4.91 (AWRT) 2/3 SC For this part, if in degrees max mark is M1 A0 SC No working shown 4 correct answers 3/3 3 correct answers 2/3 0, 1, 2 correct answers 0/3
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$e^{-2x} = 4$			
	$-2x = \ln 4$	M1		
	$x = -\frac{1}{2}\ln 4$ ISW	A1	2	OE, eg $\ln\frac{1}{2}, -\ln 2, \frac{\ln 4}{-2}$
(b)(i)	$(y=)3$	B1	1	Condone $(0,3)$ but not $(3,0)$
(ii)	$y = 0$			
	$4e^{-2x} - e^{-4x} = 0$			
	$4e^{2x} - 1 = 0$	M1		$a e^{\pm 2x} \pm b = 0$
	$e^{2x} = \frac{1}{4}$ or $e^{-2x} = 4$	A1		
	$x = \ln\frac{1}{2}$ ISW	A1	3	OE, eg $-\frac{1}{2}\ln 4, -\ln 2, \frac{1}{2}\ln\frac{1}{4}$ and no extra solutions
	Or			
	$4e^{-2x} = e^{-4x}$			
	$\ln 4 - 2x = -4x$	(M1)		
	$2x = -\ln 4$	(A1)		OE
	$x = -\frac{1}{2}\ln 4$	(A1)		OE
(iii)	$(y' =) -8e^{-2x} + 4e^{-4x}$	B1		
	$4e^{-4x} = 8e^{-2x}$			
	$2e^{2x} - 1 = 0$ or $e^{-2x} - 2 = 0$			
	or $e^{2x} = \frac{1}{2}$ or $e^{-2x} = 2$	M1		Equating $\frac{dy}{dx} = 0$ and getting $a e^{\pm 2x} \pm b = 0$ from $\frac{dy}{dx} = pe^{-2x} + qe^{-4x}$
	or $\ln 4 - 4x = \ln 8 - 2x$			
	$x = \frac{1}{2}\ln\frac{1}{2}$ ISW	A1	3	OE, eg $\frac{1}{2}(\ln 4 - \ln 8)$ and no extra solutions

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(b)(iv)	$V = \pi \int_0^{\ln 2} \left(4e^{-2x} - e^{-4x}\right)^2 dx$	B1		Must be completely correct including dx seen on this line or next line Limits, brackets and π PI from later working
	$= (\pi) \int 16e^{-4x} + e^{-8x} - 8e^{-6x} \,(dx)$	B1		Correct expansion, PI from later working
	$= (\pi)\left[-4e^{-4x} - \frac{1}{8}e^{-8x} + \frac{4e^{-6x}}{3}\right]_{(0)}^{(\ln 2)}$	B1		$\frac{16}{-4}e^{-4x}$ OE
		B1		$-\frac{1}{8}e^{-8x}$ OE
		B1		$\frac{-8}{-6}e^{-6x}$ OE may be two separate terms
	$= (\pi)\left[\left(-4e^{-4\ln 2} - \frac{1}{8}e^{-8\ln 2} + \frac{4}{3}e^{-6\ln 2}\right) - \left(-4e^{0} - \frac{1}{8}e^{0} + \frac{4}{3}e^{0}\right)\right]$	M1		Correct substitution of $x = \ln 2$ and 0 into their integrated expression $\left(\text{must be of form } ae^{-4x} + be^{-6x} + ce^{-8x}\right)$ and subtracting. PI
	$= \frac{5247}{2048}\pi$	A1	7	OE exact fraction eg $\frac{251856}{98304}\pi$
	Total		**16**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MPC3

(Specification 6360)

Pure Core 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3 – June 2011

Q	Solution	Marks	Total	Comments
1 (a)	$\frac{1}{6}$ or $\left(\frac{1}{6}, 0\right)$	B1	1	condone 0.167 AWRT
(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{1}{x}$	M1		$\frac{k}{x}$ where $k = 1$, 6 or $\frac{1}{6}$
		A1	2	$k = 1$
(c)	x / y			
	1: $\ln 6 = 1.7918$ 2: $\ln 12 = 2.4849$ 3: $\ln 18 = 2.8904$ 4: $\ln 24 = 3.1781$	M1		5+ y-values correct, either exact or correct to 3SF (rounded or truncated) or better
	5: $\ln 30 = 3.4012$ 6: $\ln 36 = 3.5835$ 7: $\ln 42 = 3.7377$	A1		all 7 y-values correct (and only these 7 values), either exact or correct to 3SF (rounded or truncated) or better
	$\text{A } = \frac{1}{3} \times 1\left[(1.7918 + 3.7377) + 4(2.4849 + 3.1781 + 3.5835) + 2(2.8904 + 3.4012)\right]$	M1		correct use of Simpson's rule on their 7 y-values, condone missing square brackets
	$= 18.4$	A1	4	CAO this value only
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$y = xe^{2x}$			
	$\left(\frac{dy}{dx} =\right) 2xe^{2x} + e^{2x}$	M1 A1 A1 ISW	3	$kxe^{2x} + le^{2x}$ where k and l are 1s or 2s $\left.\begin{array}{l} k = 2 \\ l = 1 \end{array}\right\}$ Independent of each other $(= e^{2x}(2x+1))$
(ii)	$x = 1 \Rightarrow \frac{dy}{dx} = 3e^2$	M1		correct substitution of x = 1 into their $\frac{dy}{dx}$ but must have earned M1 in part (i)
	tangent: $y - e^2 = 3e^2(x-1)$ OE	A1	2	CSO (no ISW), must have scored first 4 marks common correct answer: $y = 3e^2x - 2e^2$
(b)	$y = \frac{2\sin 3x}{1+\cos 3x}$			
	$\left(\frac{dy}{dx} =\right) \frac{(1+\cos 3x)6\cos 3x - 2\sin 3x(-3\sin 3x)}{(1+\cos 3x)^2}$	M1		$\frac{\pm p(1+\cos 3x)\cos 3x \pm q\sin 3x(\sin 3x)}{(1+\cos 3x)^2}$ where p and q are rational numbers condone poor use/omission of brackets PI by further working
	$= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{(1+\cos 3x)^2}$	A1		this line must be seen in this form (ie in terms of $\cos^2 3x$ and $\sin^2 3x$), but allow $\sin^2 3x$ replaced by $1-\cos^2 3x$ condone denominator correctly expanded
		m1		correct use of $k\sin^2 3x + k\cos^2 3x = k$ or $k\sin^2 3x = k(1-\cos^2 3x)$
	$= \frac{6\cos 3x + 6}{(1+\cos 3x)^2}$			
	$= \frac{6}{1+\cos 3x}$	A1	4	CSO
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
	note: if degrees used then no marks in (a) and (c)			
3(a)	$f(x) = \cos^{-1}(2x-1) - e^x$			or reverse
	$f(0.4) = 0.3$ $f(0.5) = -0.1$	M1		sight of ±0.3 (AWRT) **AND** ∓ 0.1 (AWRT)
	change of sign $\therefore 0.4 < \alpha < 0.5$	A1	2	CSO, note f (x) must be defined, condone $0.4 \le \alpha \le 0.5$
		(M1)		alternative method $e^{0.4} = 1.5$, $\cos^{-1}(2\times0.4 - 1) = 1.8$ $e^{0.5} = 1.65$, $\cos^{-1}(2\times0.5 - 1) = 1.57$
		(A1)		at 0.4 $e^x < \cos^{-1}(2x-1)$ at 0.5 $e^x > \cos^{-1}(2x-1)$ $\therefore 0.4 < \alpha < 0.5$
(b)	$\cos^{-1}(2x-1) = e^x$ $2x - 1 = \cos(e^x)$ $x = \frac{1}{2}\left(\cos(e^x) + 1\right) = \frac{1}{2} + \frac{1}{2}\cos(e^x)$	B1	1	AG must see middle line, and no errors seen, but condone cos e^x
(c)	$x_1 = 0.4$ $x_2 = 0.539$ $x_3 = 0.428$	B1 B1	2	CAO CAO
	Total		**5**	

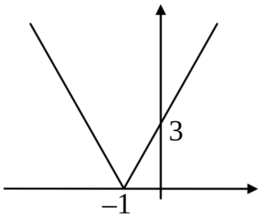
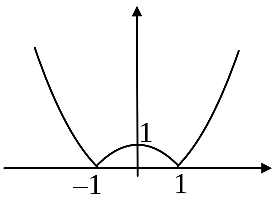
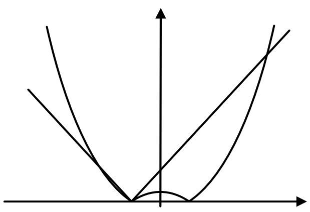
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\left(\sin^{-1} \pm 0.25 =\right) \pm 14.5$	M1		PI by sight of 194.5 etc condone ± 14.4
	$\theta = 194.5,\ 345.5$ (AWRT)	A1	2	no extras in interval, ignore answers outside interval
(ii)	$2\cot^2(2x+30) = 2 - 7\operatorname{cosec}(2x+30)$			condone replacing 2*x* + 30 by *Y*
	$2(\operatorname{cosec}^2(2x+30) - 1) = 2 - 7\operatorname{cosec}(2x+30)$	M1		correct use of $\operatorname{cosec}^2 Y = 1 + \cot^2 Y$
	$2\operatorname{cosec}^2(2x+30) + 7\operatorname{cosec}(2x+30) - 4 (= 0)$	A1		must be in this form
	$(2\operatorname{cosec}(2x+30) \pm 1)(\operatorname{cosec}(2x+30) \pm 4) (= 0)$	m1		attempt at factorisation
	$\operatorname{cosec}(2x+30) = \frac{1}{2}$ or -4	A1		must be this line using f (2*x* + 30)
	$2x + 30 = 194.5,\ 345.5$			
	$x = 82.2,\ 157.8$ (AWRT)	B1		one correct answer, allow 82.3, ignore extra solutions
		B1	6	CAO both answers correct and no extras in interval, ignore answers outside interval
(b)	stretch (I) scale factor $\frac{1}{2}$ (II) parallel to *x*-axis (III)	M1 A1		I and either II or III I + II + III
	translate	E1		
	$\begin{pmatrix} -15 \\ 0 \end{pmatrix}$	B1	4	condone '15 to left' or '–15 in *x* (direction)'
	alternative method translate	(E1)		
	$\begin{pmatrix} -30 \\ 0 \end{pmatrix}$	(B1)		
	stretch	(M1)		as above
	scale factor $\frac{1}{2}$ parallel to *x*-axis	(A1)		as above
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\left[\mathrm{f}(x)\right]$ not $1-1$	E1	1	OE
(b)	$y=\dfrac{1}{2x+1}$			
	$x=\dfrac{1}{2y+1}$	M1		swap x and y } either order
	$2y+1=\dfrac{1}{x}$	M1		a correct next line } either order
	$\left[g^{-1}(x)=\right]\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)$ OE	A1	3	$\left[y=\right]\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)$
(c)	$\left[\mathrm{g}^{-1}(x)\right]\neq -0.5$	B1	1	sight of $\neq -0.5$ OE
(d)	$\left(\dfrac{1}{2x+1}\right)^2=\dfrac{1}{2x+1}$	B1		sight of $\left(\dfrac{1}{2x+1}\right)^2$ or $\dfrac{1}{(2x+1)^2}$
	$(2x+1)=(2x+1)^2$ or $2x+1=4x^2+4x+1$ or $\dfrac{1}{2x+1}=1$ or $2x + 1 = 1$	M1		one correct step, must be one of these four lines
	$x=0$	A1	3	CSO
	Total		**8**	
6(a)	$3\ln x=4$			
	$\left(\ln x=\dfrac{4}{3}\right)$			
	$x=\mathrm{e}^{\frac{4}{3}}$	B1	1	ISW. Condone $\sqrt[3]{\mathrm{e}^4}$
(b)	$3\ln x+\dfrac{20}{\ln x}=19$			
	$3(\ln x)^2+20=19\ln x$	M1		correctly multiplying by ln x.
	$3(\ln x)^2-19\ln x+20(=0)$	A1		
	$(3\ln x\pm 4)(\ln x\pm 5)(=0)$	m1		use of formula, or completing the square must be correct
	$\ln x=\dfrac{4}{3},\ 5$	A1		
	$x=\mathrm{e}^{\frac{4}{3}},\ \mathrm{e}^5$	A1	5	condone $\sqrt[3]{\mathrm{e}^4}$
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)		M1 A1	 2	modulus graph, approximate V shape, touching negative x-axis and crossing y-axis -1, 3 marked, graph symmetrical, straight lines
(ii)		M1 A1 A1	 3	modulus graph in 3 sections, touching x-axis and crossing positive y-axis correct curvature their $x > 1$, their $x < -1$ } independent correct curve $-1 \le x \le 1$ and $x = \pm 1$, $y = 1$ marked
(b)(i)	$\lvert 3x+3 \rvert = \lvert x^2 - 1 \rvert$ $(3x+3 = x^2 - 1)$ $(0 =)\ x^2 - 3x - 4$ —A $x = 4, -1$ $(3x + 3 = 1 - x^2)$ $x^2 + 3x + 2\ (= 0)$ —B $x = -1, -2$	 M1 A1,A1 A1,A1	 5	 either A or B seen, all terms on one side $\therefore\ x = -2, -1, 4$ SC NMS or partial method 1 correct value 1/5 2 correct values 2/5 3 correct values 5/5 } independent of method mark more than 3 distinct values max 2/5
(ii)	 $x > 4$, $x < -2$	M1,A1	2	$x >$ their largest, $x <$ their smallest; CAO
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8	$\int \frac{1}{\cos^2 x(1+2\tan x)^2}\,\mathrm{d}x$			
	$u = 1 + 2\tan x$			
	$\left(\frac{\mathrm{d}u}{\mathrm{d}x} =\right) 2\sec^2 x$ OE	M1		condone $\left(\frac{\mathrm{d}u}{\mathrm{d}x} =\right) a\sec^2 x$ where a is a constant
	$\int = \int \frac{\mathrm{d}u}{2u^2}$	m1		$\int \frac{k}{u^2}\ (\mathrm{d}u)$, where k is a constant
		A1		correct, or $\frac{1}{2}\int u^{-2}\,(\mathrm{d}u)$
	$= \frac{1}{2}\frac{u^{-1}}{-1}$	A1F		correct integral of their expression but must have scored M1 m1
	$= -\frac{1}{2u}$			
	$= -\frac{1}{2(1+2\tan x)}(+c)$	A1	5	CSO, no ISW
	Total		**5**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9 (a)	$\int x \ln x \, dx$			
	$u = \ln x \quad \frac{dv}{(dx)} = x$; $\frac{du}{(dx)} = \frac{1}{x} \quad v = \frac{x^2}{2}$	M1		correct direction and sight of $\frac{1}{x}, \frac{x^2}{2}$
	$\int = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{1}{x}\,(dx)$	A1		
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4}(+c)$	A1	3	
(b)	$y = (\ln x)^2$			
	$\left(\frac{dy}{dx}\right) = 2\ln x \times \frac{1}{x}$	M1		$\frac{k}{x}\ln x$ where $k = \frac{1}{2}, 1$ or 2
		A1	2	$k = 2$
(c)	$y = \sqrt{x}\,\ln x$			
	$(V =)\pi \int_1^e x(\ln x)^2\,dx$	B1		all correct, incl brackets, π, limits and dx (but dx may be seen BEFORE this line)
	$u = (\ln x)^2 \quad \frac{dv}{(dx)} = x$; $\frac{du}{(dx)} = 2\ln x \frac{1}{x} \quad v = \frac{x^2}{2}$	M1		correct direction with $\frac{du}{(dx)} = \frac{k}{x}\ln x$ where $k = \frac{1}{2}, 1$ or 2 and sight of $\frac{x^2}{2}$
	$\int = \frac{x^2}{2}(\ln x)^2 - \int \frac{x^2}{2} \times \frac{2}{x}\ln x\,(dx)$	m1		correct substitution of their terms into the parts formula
	$= \frac{x^2}{2}(\ln x)^2 - \int x \ln x\,(dx)$	A1		integral needs to be simplified to $\int x \ln x$
	$= \frac{x^2}{2}(\ln x)^2 - \frac{1}{4}x^2(2\ln x - 1)$ OE			
	$V = (\pi)\left[\frac{x^2}{2}(\ln x)^2 - \frac{1}{4}x^2(2\ln x - 1)\right]_1^e$			
	$= (\pi)\left[\left(\frac{e^2}{2} - \frac{1}{4}e^2\right) - \left(0 + \frac{1}{4}\right)\right]$	m1		correct substitution of 1 and e into their expressions of the form $px^2(\ln x)^2 + qx^2\ln x + rx^2$ where p, q and r are non-zero rational numbers, and an intention to subtract. Do not condone F(1) – F(e)
	$= \frac{\pi}{4}\left[e^2 - 1\right]$ **OE**	A1	6	$\pi\left[\frac{e^2}{4} - \frac{1}{4}\right]$ etc
	Total		**11**	
	TOTAL		**75**	

General Certificate of Education (A-level)
January 2012

Mathematics MPC3

(Specification 6360)

Pure Core 3

Final

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	x | y 0 | 1 $\frac{1}{2}$ | 2 1 | 4 $1\frac{1}{2}$ | 8 2 | 16 $2\frac{1}{2}$ | 32 3 | 64	B1 B1		all 7 x values correct (and no extra) (PI by 7 correct y values) 5 or more correct y values, exact $\left(4^{\frac{1}{2}}, 4^{1}...\right)$ or evaluated (in table or in formula)
	$A = \frac{1}{3}\times\frac{1}{2}[\ 65 + 4\times 42 + 2\times 20\ ]$	M1		correct substitution of their 7 y-values into Simpson's rule
	$= \frac{91}{2}$ or 45.5 or $\frac{273}{6}$	A1	4	CAO
(b)(i)	$\text{f}(x) = 4^x + 2x - 8$ or $\text{g}(x) = 8 - 2x - 4^x$ $\left.\begin{array}{l}\text{f}(1.2) = -0.3 \quad \text{or} \quad \text{g}(1.2) = 0.3\\ \text{f}(1.3) = 0.7 \quad \text{or} \quad \text{g}(1.3) = -0.7\end{array}\right\}$ AWRT ± 0.3 and ± 0.7 condone $\text{f}(1.2) < 0$, $\text{f}(1.3) > 0$ if f is defined	M1		attempt at evaluating f(1.2) and f(1.3) **alternative method** $\left.\begin{array}{l}4^{1.2} = 5.3,\ 8 - 2\times 1.2 = 5.6\\ 4^{1.3} = 6.1,\ 8 - 2\times 1.3 = 5.4\end{array}\right\}$ M1
	change of sign $\therefore\ 1.2 < \alpha < 1.3$ (f(x) must be defined and all working correct)	A1	2	at 1.2 LHS < RHS at 1.3 LHS > RHS $\therefore\ 1.2 < \alpha < 1.3$ A1
(ii)	$(x_2 =)\ 1.243$ $(x_3 =)\ 1.232$	B1 B1	 2	 these values only
	Total		**8**	

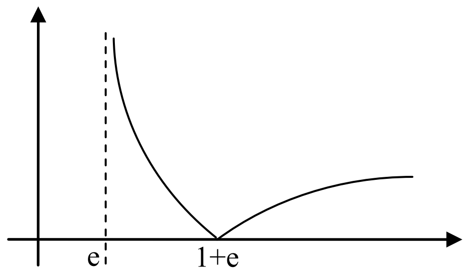
MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\left.\begin{array}{l} f(1)=21 \\ f(16)=1 \end{array}\right\}$	M1		sight of 1 and 21
	$1 \le f(x) \le 21$	A1	2	allow f(x) replaced by f, y
(b)(i)	$y=\dfrac{63}{4x-1}$			
	$x=\dfrac{63}{4y-1}$	M1		reverse x, y } Either order
	$x(4y-1)=63$ or better	M1		one correct step } Either order
	$f^{-1}(x)=\dfrac{1}{4}\left(\dfrac{63}{x}+1\right)$ OE	A1	3	condone $y=$
(ii)	$\dfrac{1}{4}\left(\dfrac{63}{x}+1\right)=1$			
	$\dfrac{63}{x}+1=4$, or better	M1		one correct step from their (b)(i) = 1, or $x=$ f(1)
	$(x=)\ 21$	A1	2	note: 21 scores 2/2
(c)(i)	$\left(\text{fg}(x)=\right)\dfrac{63}{4x^2-1}$	B1	1	
(ii)	$\dfrac{63}{4x^2-1}=1$			
	$4x^2-1=63$ or better	M1		one correct step from their (c)(i) = 1
	$x^2=16$ OE	A1		eg $(2x+8)(2x-8)=0$, or $x=\pm 4$
	$x=-4$ **ONLY**	A1	3	
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ 12x^2-6$	B1	1	do not ISW
(b)	$\int_2^3 \frac{2x^2-1}{4x^3-6x+1}\,\mathrm{d}x$			
	$=\left[\frac{1}{6}\ln\left(4x^3-6x+1\right)\right]_{(2)}^{(3)}$	M1		$k\ln\left(4x^3-6x+1\right), k$ is a constant
		A1		$k=\frac{1}{6}$
	$=\frac{1}{6}\ln\left(4\times3^3-6\times3+1\right)$ $-\frac{1}{6}\ln\left(4\times2^3-6\times2+1\right)$	m1		correct substitution in F(3) – F(2). condone poor use or lack of brackets.
	$=\frac{1}{6}\ln 91-\frac{1}{6}\ln 21$	A1F		$k\ln 91-k\ln 21$ only follow through on their k
	$=\frac{1}{6}\ln\frac{91}{21}$ or $\left(=\frac{1}{6}\ln\frac{13}{3}\right)$	A1	5	
				or if using the substitution $u=4x^3-6x+1$ $\int = k\int\frac{\mathrm{d}u}{u}$ M1 $=\frac{1}{6}\ln u$ A1 then, either change limits to 21 and 91 m1 then A1F A1as scheme or changing back to 'x', then m1 A1F A1 as scheme
	Total		**6**	
4(a)	$\sec^2\theta-1=...$	B1		correct use of $\sec^2\theta=1+\tan^2\theta$
	$\sec^2\theta+3\sec\theta-10\ (=0)$	M1		quadratic expression in $\sec\theta$ with all terms on one side
	$(\sec\theta+5)(\sec\theta-2)=0$	m1		attempt at factors of their quadratic, $(\sec\theta\pm5)(\sec\theta\pm2)$, or correct use of quadratic formula
	$\sec\theta=-5,\ 2$ $\left(\cos\theta=-\frac{1}{5},\frac{1}{2}\right)$	A1		
	$60^\circ,\ 300^\circ, 101.5^\circ,\ 258.5^\circ$ (AWRT)	B1 B1	6	3 correct, ignore answers outside interval all correct, no extras in interval
(b)	$4x-10^\circ=60^\circ, 101\cdot5^\circ, 258\cdot5^\circ, 300^\circ$	M1		$4x-10=$ any of their (60),
	$4x=70^\circ, 111\cdot5^\circ, 268\cdot5^\circ, 310^\circ$	A1F		all their answers from (a), BUT must have scored B1
	$x=17\cdot5^\circ, 27\cdot9^\circ, 67.1^\circ, 77\cdot5^\circ$ (AWRT)	A1	3	CAO, ignore answers outside interval
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	stretch (I); SF 4 (II); in y-direction (III) — either order	M1A1		I + (II or III)
	translate	E1		
	$\begin{pmatrix} e \\ 0 \end{pmatrix}$	B1	4	accept 'e in positive x-direction'
(b)		M1		mod graph, in 2 connected sections, both in the first quadrant, touching x-axis
		A1		curve touches x-axis at 1 + e (or 3.7 or better), and labelled (ignore scale)
		A1	3	correct curvature, including at their 1+ e, approx. asymptote at x = e
(c)(i)	$\lvert 4\ln(x-e)\rvert = 4$			
	$4\ln(x-e)=4$ $\quad 4\ln(x-e)=-4$ or better	M1		must see 2 equations, condone omission of brackets
	$(x =)$ 2e do not ISW	A1		accept values of AWRT 5.42, 5.43, 5.44
	$(x=)e+e^{-1}$ or $(x=)e+\frac{1}{e}$ do not ISW	A1	3	accept values of AWRT 3.08, 3.09 if M0 then x = 2e with or without working scores SC1
(ii)	$x \geq 2e$	B1		accept values of AWRT 5.42, 5.43, 5.44
	$e < x \leq e + \frac{1}{e}$	B2	3	accept values of AWRT 2.72, 3.08, 3.09 if B2 not earned, then SC1for any of $e \leq x \leq e+\frac{1}{e}$, $e < x < e+\frac{1}{e}$, $e \leq x < e+\frac{1}{e}$
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\left(\dfrac{dx}{d\theta}=\right)\dfrac{(\sin\theta\times 0)-1\times\cos\theta}{\sin^2\theta}$	M1		quotient rule $\dfrac{\pm\sin\theta\times k\pm 1\times\cos\theta}{\sin^2\theta}$ where $k=0$ or 1
		A1		must see the '0' either in the quotient or in eg $\dfrac{du}{d\theta}=0$ etc
	$=-\dfrac{\cos\theta}{\sin^2\theta}$ or $=-\dfrac{\cos\theta}{\sin\theta\sin\theta}$			or equivalent
	$=-\operatorname{cosec}\theta\cot\theta$	A1	3	CSO, AG must see one of the previous expressions
(b)	$x=\operatorname{cosec}\theta$			
	$\dfrac{dx}{d\theta}=-\operatorname{cosec}\theta\cot\theta$	B1		OE, eg $dx=-\operatorname{cosec}\theta\cot\theta\, d\theta$
	Replacing $\sqrt{(\operatorname{cosec}^2\theta-1)}$ by $\sqrt{\cot^2\theta}$, or better	B1		at any stage of solution
	$\int=\int\dfrac{-\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta\sqrt{(\operatorname{cosec}^2\theta-1)}}d\theta$	M1		all in terms of θ, and including their attempt at dx, but condone omission of $d\theta$
		A1		fully correct and must include $d\theta$ (at some stage in solution)
	$\int\dfrac{-\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta\cot\theta}(d\theta)$			
	$=\int\dfrac{-1}{\operatorname{cosec}\theta}(d\theta)$	A1		OE eg $\int -\sin\theta(d\theta)$
	$=\cos\theta$	A1		
	$x=2,\ \theta=0.524$ AWRT $x=\sqrt{2},\ \theta=0.785$ AWRT	B1		correct change of limits
				or $(\pm)\cos\theta=(\pm)\left[\sqrt{\left(1-\dfrac{1}{x^2}\right)}\right]_{\sqrt{2}}^{2}$ OE
	$0.8660-0.7071$	m1		c's $F(0.52)-F(0.79)$ substitution into $\pm\cos\theta$ only or $\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{\sqrt{2}}\right)$
	$=0.159$	A1	9	
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\left[\frac{dy}{dx}=\right] p\,e^{-\frac{1}{4}x}x^2+qxe^{-\frac{1}{4}x}$	M1 A1		p, q constants $p=-\frac{1}{4}$ **and** $q=2$
	$\left[\Rightarrow e^{-\frac{1}{4}x}\left(-\frac{1}{4}x^2+2x\right)=0\right]$			
	$e^{-\frac{1}{4}x}\neq 0$	E1		or $e^{-\frac{1}{4}x}=0$ impossible OE (may be seen later)
	$\left(e^{-\frac{1}{4}x}\right)\left(ax^2+bx\right)=0$	m1		or $e^{-\frac{1}{4}x}x\left(ax+b\right)=0$
	$x=0, 8$	A1		
	$x=0,\ y=0$	A1		
	$x=8,\ y=64e^{-2}$	B1	7	condone $y=8^2e^{-\frac{8}{4}}$ etc ignore further numerical evaluation
(b)(i)	$\int x^2e^{-\frac{1}{4}x}\,dx \qquad u=x^2 \qquad \frac{dv}{dx}=e^{-\frac{1}{4}x}$ $\frac{du}{dx}=2x \qquad v=ke^{-\frac{1}{4}x}$	M1		where k is a constant
	$k=-4$	A1		
	$-4x^2e^{-\frac{1}{4}x}-\int -4e^{-\frac{1}{4}x}\times 2x(dx)$, or better	A1F		correct substitution of their terms
	$u=mx \qquad \frac{dv}{dx}=ne^{-\frac{1}{4}x}$ $\frac{du}{dx}=m \qquad v=-4ne^{-\frac{1}{4}x}$	m1		both differentiation and integration **must** be correct
	$\int=-4x^2e^{-\frac{1}{4}x}+8\left(-4xe^{-\frac{1}{4}x}+\int 4e^{-\frac{1}{4}x}dx\right)$			
	$=\left[-4x^2e^{-\frac{1}{4}x}-32xe^{-\frac{1}{4}x}-128e^{-\frac{1}{4}x}\right]_{(0)}^{(4)}$	A1		
	$=-e^{-1}\left[64+256\right]-\left[-128\right]$	m1 (dep on M1 only)		correct substitution and attempt at subtraction in $ax^2e^{-\frac{1}{4}x}+bxe^{-\frac{1}{4}x}+ce^{-\frac{1}{4}x}$ (may be in 3 stages)
	$=128-\frac{320}{e}$	A1	7	or $128-320e^{-1}$ ignore further numerical evaluation
(ii)	$v=\pi\int_{(0)}^{(4)}9x^2e^{-\frac{1}{4}x}\left(dx\right)$	M1		condone omission of brackets, limits
	$=9\pi\left(128-\frac{320}{e}\right)$	A1F	2	$9\pi\times$ (their exact b(i))
	Total		**16**	
	TOTAL		**75**	

Version 1,0

General Certificate of Education (A-level)
June 2012

Mathematics MPC3

(Specification 6360)

Pure Core 3

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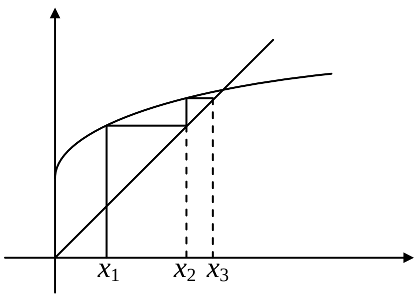
Otherwise we require evidence of a correct method for any marks to be awarded.

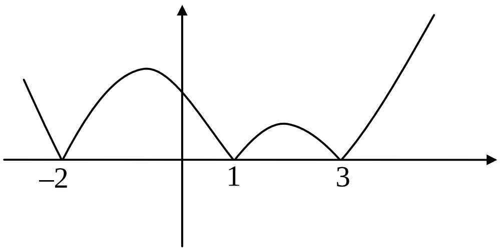
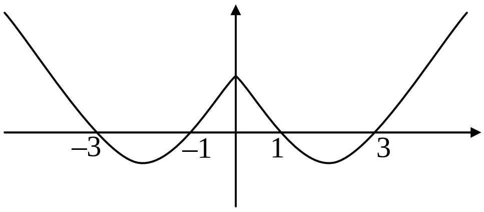
Q	Solution	Marks	Total	Comments
1	<table><tr><th>x</th><th>y</th></tr><tr><td>0.5</td><td>3.9163</td></tr><tr><td>0.7</td><td>1.8748</td></tr><tr><td>0.9</td><td>0.9520</td></tr><tr><td>1.1</td><td>0.3773</td></tr></table>	B1		All 4 correct x values (and no extras used)
		M1		3+ y decimal values rounded or truncated to 2 dp or better (in table or in formula) (PI by correct answer)
	$\int = 0.2 \times \sum y$	m1		Correct substitution of their 4 y values (of which 3 are correct), either listed or totalled
	$(= 0.2 \times 7.12\ldots)$ $= 1.424$	A1	4	CAO
	Total		**4**	

Q	Solution	Marks	Total	Comments
2(a)	$f(x) = 4\ln x - \sqrt{x}$			Or reverse
	$\left.\begin{array}{l} f(0.5) = -3.5 \\ f(1.5) = 0.4 \end{array}\right\}$ must have both values correct	M1		Allow $f(0.5) < 0$ and $f(1.5) > 0$ only if f(*x*) defined
	Change of sign $\therefore\ 0.5 < \alpha < 1\cdot5$	A1	2	f(*x*) must be defined and all working correct, including both statement and interval (either may be written in words or symbols) **OR** comparing 2 sides: $\left.\begin{array}{ll} 4\ln 0.5 = -2.8 & \sqrt{0.5} = 0.7 \\ 4\ln 1.5 = 1.6 & \sqrt{1.5} = 1.2 \end{array}\right\}$ (M1) At 0.5, LHS < RHS; at 1.5, LHS > RHS $\therefore 0.5 < \alpha < 1.5$ (A1)
(b)	$\ln x = \dfrac{\sqrt{x}}{4}$ or $x^4 = e^{\sqrt{x}}$			Must be seen
	$x = e^{\frac{\sqrt{x}}{4}}$	B1	1	AG; no errors seen
(c)	$x_2 = 1.193$	B1		
	$x_3 = 1.314$	B1	2	If B0B0 scored but either value seen correct to 2 or 4 dp, score SC1
(d)		M1		Vertical line from x_1 to curve (condone omission from *x*-axis to *y* = *x*) and then horizontal to *y* = *x*
		A1	2	2[nd] vertical and horizontal lines, and x_2, x_3 (not the values) must be labelled on *x*-axis
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{dy}{dx}=\right)\quad x^3\times\frac{1}{x}+3x^2\ln x$	M1		$px^3\times\frac{1}{x}+qx^2\ln x$ where p and q are integers
		A1	2	$p=1, q=3$
(b)(i)	$\left(\frac{dy}{dx}=\right)\quad e^2+3e^2\ln e\quad\left(=4e^2\right)$	M1		Substituting e for x in their $\frac{dy}{dx}$, but must have scored M1 in (a)
	$y=e^3\ln e\ \left(=e^3\right)$	B1		
	$y-e^3=4e^2\left(x-e\right)$	A1	3	OE **but** must have evaluated ln e (twice) for this mark (must be in exact form, but condone numerical evaluation after correct equation)
(ii)	$-e^3=4e^2\left(x-e\right)$ or $4e^2x=3e^3$ OE	M1		Correctly substituting $y = 0$ into a correct tangent equation in (b)(i)
	$x=\frac{3}{4}e$	A1	2	CSO; ignore subsequent decimal evaluation
	Total		**7**	
4(a)	$\int xe^{6x}\,dx$			
	$u=x\quad\frac{dv}{(dx)}=e^{6x}$	M1		All 4 terms in this form, $k=\frac{1}{6}$, 1 or 6
	$\frac{du}{(dx)}=1\quad v=ke^{6x}$	A1		$k=\frac{1}{6}$
	$\frac{1}{6}xe^{6x}-\int\frac{1}{6}e^{6x}\left(dx\right)$	A1F		Correct substitution of their terms into parts formula
	$=\frac{1}{6}xe^{6x}-\frac{1}{36}e^{6x}\left(+c\right)$ OE	A1	4	No ISW for incorrect simplification
(b)	$\left(V=\right)\quad\pi\int_0^1 xe^{6x}\,dx$	B1		Must include π, limits and dx
	$=\left(\pi\right)\left[\left(\frac{1}{6}e^6-\frac{1}{36}e^6\right)-\left(-\frac{1}{36}\right)\right]$	M1		Correct substitution of 0 and 1 into their answer in (a), must be of the form $axe^{6x}-be^{6x}$, where $a > 0$, $b > 0$ and F(1) – F(0) seen
	$=\pi\left[\frac{5}{36}e^6+\frac{1}{36}\right]$	A1	3	CAO; ISW
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)	$\text{f}(x) \geq 0$	M1 A1	2	$\text{f}(x) > 0,\ \text{f} \geq 0,\ x \geq 0,\ y > 0,\ \text{range} \geq 0$ Condone $y \geq 0$
(b)(i)	$\text{fg}(x) = \sqrt{2\left(\frac{10}{x}\right) - 5}$ $\left(= \sqrt{\frac{20}{x} - 5}\right)$ OE	B1	1	No ISW
(ii)	$\sqrt{\frac{20}{x} - 5} = 5$			
	$\frac{20}{x} = 5^2 + 5$	M1		Correctly squaring their fg(x) and correctly isolating their x term
	$x = \frac{2}{3}$	A1	2	No ISW
(c)(i)	$y = \sqrt{2x - 5}$			
		M1 M1		Swap x and y } either order Correctly squaring } either order
	$\left(\text{f}^{-1}(x) =\right) \frac{x^2 + 5}{2}$	A1	3	
(ii)	$x^2 = 9$ or if $\sqrt{9}$ or 3 seen	M1		Candidate must have scored full marks in (c)(i) (ie no follow through)
	$x = 3$ and $x = -3$ rejected	A1	2	Must see both
	Total		**10**	

Q	Solution	Marks	Total	Comments
6	$u = x^4 + 2$			
	$\frac{du}{dx} = 4x^3$	B1		or $du = 4x^3\ dx$
	$\int \frac{x^7}{(x^4+2)^2}\ dx$			
	$= \int \frac{k(u-2)}{u^2}\ du$ or $\int \frac{k(u-2)^{\frac{7}{4}}}{u^2} \frac{du}{(u-2)^{\frac{3}{4}}}$	M1		Either expression all in terms of u including replacing dx, but condone omission of du
	$= \left(\frac{1}{4}\right) \int \frac{1}{u} - \frac{2}{u^2}\ du$	m1		$k\int au^{-1} + bu^{-2}\,du$, where k, a, b are constants
	$= \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]$	A1		Must have seen du on an earlier line where every term is a term in u
	$\left(\int \quad = \left(\frac{1}{4}\right)\left[\ln u + \frac{2}{u}\right]_2^3\right)$			$\left(\left(\frac{1}{4}\right)\left[\ln\left(x^4+2\right) + \frac{2}{\left(x^4+2\right)}\right]_0^1\right)$
	$= \left(\frac{1}{4}\right)\left[\left(\ln 3 + \frac{2}{3}\right) - \left(\ln 2 + 1\right)\right]$	m1		Dependent on previous A1 Correct change of limits, correct substitution and F(3) – F(2) or correct replacement of u, correct substitution and F(1) – F(0)
	$= \frac{1}{4}\ln\left(\frac{3}{2}\right) - \frac{1}{12}$	A1	6	OE in exact form
	Total		**6**	

Q	Solution	Marks	Total	Comments
7(a)		M1		Modulus graph, 4 sections touching x-axis at $-2, 1, 3$
		A1		Correct $x > 3$, $x < -2$
		A1	3	Correct $-2 \le x \le 3$ with maximum at 2 lower than maximum at -1 and correct cusps at $x = -2$, $x = 1$ and $x = 3$ The maximums need to be at $x = -1$ and 2 (approx)
(b)		M1		Symmetrical about y-axis, from original curve for $0 < x < 1$ and $x > 3$
		A1	2	Correct graph including cusp at $x = 0$
(c)	Translate $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	E1 B1		
	Stretch (I) sf $\frac{1}{2}$ (II) // y-axis (III) either order	M1		I and (either II or III)
		A1	4	I + II + III
(d)	$x = -2$	B1		
	$y = 5$	B1	2	Each value may be stated or shown as coordinates
	Total		**11**	

Q	Solution	Marks	Total	Comments
8(a)	LHS = $\dfrac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$	M1		Combining fractions
	$= \dfrac{2}{1-\cos^2\theta}$	A1		Correctly simplified
	$= \dfrac{2}{\sin^2\theta}$	m1		Use of $\sin^2\theta+\cos^2\theta=1$
	$2\,\text{cosec}^2\theta = 32$ $\text{cosec}^2\theta = 16$	A1	4	AG; no errors seen **OR** $1-\cos\theta+1+\cos\theta = 32(1+\cos\theta)(1-\cos\theta)$ (M1) $2=32(1-\cos^2\theta)$ (A1) $2=32\sin^2\theta$ (m1) $\text{cosec}^2\theta=16$ (A1)
(b)	$\text{cosec}\, y = (\pm)\sqrt{16}$ or better (PI by further working)	M1		or $\sin y = (\pm)\sqrt{\dfrac{1}{16}}$ or better
	(y =) 0.253, (2.889,) (3.394,) (6.031,) (–0.253)	B1		Sight of any of these correct to 3dp or better
	(y =) 0.25, 2.89, 3.39 (or better)	A1		Must see these 3 answers, with or without either/both of −0.25 or 6.03 Ignore answers outside interval −0.25 to 6.03 but extras in this interval scores A0
	x = 0.43, 1.74, 2(.00), 0.17	B1 B1	5	3 correct (must be 2 dp) All 4 correct (must be 2 dp) and no extras in interval (ignore answers outside interval)
	Total		**9**	

Q	Solution	Marks	Total	Comments
9(a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}y}=\right) \quad \frac{\cos y \times \cos y - \sin y \times -\sin y}{\cos^2 y}$	M1		Condone incorrect signs, poor notation, omission of $\frac{\mathrm{d}x}{\mathrm{d}y}$ or using $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$= \frac{\cos^2 y + \sin^2 y}{\cos^2 y}$	A1		RHS correct with terms squared, including correct notation Must see this line
	$= \frac{1}{\cos^2 y}$ or $\left(=1+\tan^2 y\right)$			Must see one of these
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \sec^2 y$	A1 CSO	3	AG; all correct including correct use of $\frac{\mathrm{d}x}{\mathrm{d}y}$ throughout
(b)	$\sec^2 y = 1+\left(x-1\right)^2$	M1		Correct use of $\sec^2 y = 1+\tan^2 y$ and in terms of x
	$= 1+x^2-2x+1$ OE			
	$= x^2-2x+2$	A1	2	AG; must see "$\sec^2 y =$", $(x-1)^2$ expanded and no errors seen
(c)	$\frac{\mathrm{d}x}{\mathrm{d}y} = x^2-2x+2$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sec^2 y}$			Must be seen
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2-2x+2}$	B1	1	AG and no errors seen

Q	Solution	Marks	Total	Comments
9 cont (d)(i)	$y = \tan^{-1}(x-1) - \ln x$			
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\ \frac{1}{x^2-2x+2} - \frac{1}{x}$	M1		Must be correct
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} = 0\right)$			
	$\pm x^2 + bx + c\ (=0)$	m1		Expression in this form (generous), where b and $c \neq 0$
	$x^2 - 3x + 2 = 0$	A1		Must see correct equation = 0
	$x = 1,\ 2$	A1	4	Both answers must be seen The two A marks are independent
(ii)		M1		$y'' = p\left(x^2-2x+2\right)^{-2}(2x-2) \pm qx^{-2}$ where p and q are constants
	$y'' = -\left(x^2-2x+2\right)^{-2}(2x-2) + x^{-2}$	A1	2	$p = -1$, $q = 1$ including correct brackets
(iii)	$x = 1,\ y'' = 1$	M1		Must have scored full marks in (d)(i) and (ii)
	At $x=1$, $y''>0$ $\therefore$ min When $x = 1$, $y = 0$ hence on x-axis	A1	2	Must see $y''>0$ or in words Both statements fully correct
	Total		**14**	
	TOTAL		**75**	

Version

General Certificate of Education (A-level)
January 2013

Mathematics **MPC3**

(Specification 6360)

Pure Core 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

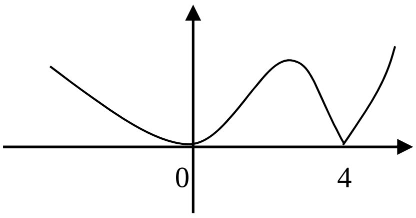
Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$f(2) = -3$ $f(3) = 10$	M1		$f(x) = x^3 - 6x + 1$ must have both values correct allow $f(2) < 0$ and $f(3) > 0$ only if $f(x)$ is defined and no errors seen
	change of sign $\Rightarrow 2 < \alpha < 3$	A1	2	must have **both statement and interval** which may be written in words/symbols
(b)	$x^3 = 6x - 1$ or $x^2 - 6 + \frac{1}{x} = 0$ or $x^2 - 6 = -\frac{1}{x}$			must see one of these lines and no errors
	$x^2 = 6 - \frac{1}{x}$	B1	1	AG
(c)	$x_2 = \sqrt{6 - \frac{1}{2.5}} = 2.366(432)$	B1		at least 4sf needed PI by correct x_3
	$x_3 = 2.362$	B1	2	SC1 if B0B0 scored and x_3 = 2.3617
	Total		**5**	

Q	Solution	Marks	Total	Comments
2(a)	$y(0)=0$ $y(1)=\frac{1}{3}=0.\dot{3}$ $y(2)=\frac{1}{3}=0.\dot{3}$	B1		all 5 *x*-values PI by 5 correct *y*-values
	$y(3)=\frac{3}{11}=0.\dot{2}\dot{7}$ $y(4)=\frac{4}{18}=0.\dot{2}$	B1		at least 4 *y*-values exact or rounded or truncated to at least 4sf
	$\frac{1}{3}\times 1\left(0+0.\dot{2}+4\left[0.\dot{3}+0.\dot{2}\dot{7}\right]+2\left[0.\dot{3}\right]\right)$	M1		correct use of Simpson's rule using $\frac{1}{3}$ and 4 and 2 correctly with candidate's 5 *y*-values
	= 1.104	A1	4	CAO (must be exactly this value)
(b)	$\int_0^4 \frac{x}{x^2+2}\,dx=\frac{1}{2}\left[\ln\left(x^2+2\right)\right]$	M1 A1		for $k\ln\left(x^2+2\right)$ all correct; limits not needed
	$=\frac{1}{2}(\ln 18-\ln 2)$	A1F		For *k* (ln18 – ln2)
	$=\frac{1}{2}\ln 9$	A1F		combining candidate's logarithms correctly (must be seen)
	$=\ln 3$	A1	5	CAO (must be exactly this) NMS scores 0/5
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{dy}{dx}=\right)3e^{3x}+\frac{1}{x}$	B1 B1	2	B1 for one term correct B1 all correct
(b)(i)	$\left(\frac{du}{dx}=\right)\frac{\pm\cos x(1+\cos x)\pm\sin x(\sin x)}{(1+\cos x)^2}$	M1		clear attempt at quotient/product rule condone poor use of brackets
	$\frac{\cos x(1+\cos x)-\sin x(-\sin x)}{(1+\cos x)^2}$	A1		any correct form seen
	$=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$			
	$=\frac{\cos x+1}{(1+\cos x)^2}$			
	$=\frac{1}{1+\cos x}$	A1cso	3	AG be convinced correct use of brackets and correct notation used throughout (eg A0 if $\cos x^2$ etc seen)
(ii)	$\left(\frac{dy}{dx}=\right)\frac{1+\cos x}{\sin x}\times\frac{1}{1+\cos x}$ OE	M1		correct use of chain rule
	$=\frac{1}{\sin x}$			
	$=\operatorname{cosec} x$	A1	2	AG, must see $=\frac{1}{\sin x}$ and no errors seen; condone incorrect use of brackets only if penalised in part (b)(i)
	Total		**7**	

Q	Solution	Marks	Total	Comments
4(a)		M1		reflection in the *x*-axis for the negative $\mathrm{f}(x)$ and remainder as given on sketch
		A1	2	correct curvatures, correct cusp at $x = 4$ condone straight lines for $x < 0$ and $x > 4$ 4 marked on *x*-axis
(b)	**Either**			
	1. Stretch 2. ‖ *x*-axis	M1		1 and either 2 or 3
	3. by factor 0.5	A1		1, 2 and 3
	(followed by) translation	E1		
	$\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$	B1	4	
	or			
	translation	(E1)		
	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	(B1)		
	(followed by) 1. Stretch 2. ‖ *x*-axis	(M1)		1 and either 2 or 3
	3. by factor 0.5	(A1)		1, 2 and 3
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)		M1		$\text{f}(x) > -\frac{4}{3}$, $\text{f} \geq -\frac{4}{3}$, range $\geq -\frac{4}{3}$
	$\text{f}(x) \geq -\frac{4}{3}$	A1	2	
(b)(i)	$x \geq -\frac{4}{3}$	B1F	1	correct or FT from **(a)**
(ii)	$x^2 = 3y + 4$			
	$x = (\pm)\sqrt{3y+4}$	M1		either order – M1 for correctly changing the subject or reversing operations; M1 for replacing *y* with *x*
	$\left(\text{f}^{-1}(x) =\right)(-)\sqrt{3x+4}$	M1		
	$\left(\text{f}^{-1}(x) =\right) -\sqrt{3x+4}$	A1	3	(dependent on both M1 marks) correct sign
(c)(i)	$3x - 1 = 1$	M1		Or $3x - 1 = \text{e}^0$ or $3x - 1 = \pm 1$
	$\frac{2}{3}$ OE	A1	2	CAO, NMS $\frac{2}{3}$ OE scores 2/2
(ii)	g has **NO** inverse because two values of *x* map to one value (of *y*) **or** it is many-one **or** it is not one-one **or** 'it is two-one'	B1	1	must indicate no inverse with valid reason; do not accept contradictory reasons
(iii)	$\ln\left\|3 \times \frac{x^2 - 4}{3} - 1\right\|$	M1		
	$\ln\left\|x^2 - 5\right\|$	A1	2	NMS scores 0/2, condone *k* = – 5 after correct expression seen
(iv)	$\ln\left\|x^2 - 5\right\| = 0$			
	$\left\|x^2 - 5\right\| = 1$			
	$x^2 - 5 = 1$ (or – 1 or e^0 or $-\text{e}^0$ seen)	M1		$x^2 - k = 1$ etc, for candidate's positive integer, *k*
	$x^2 = 6, 4$ or candidate's $k+1$ or $k-1$			
	$x = \sqrt{6}$, 2	A1F		exact values PI by correct answers
	$x = -\sqrt{6}, -2$	A1F		
	$(x \leq 0 \Rightarrow)$ $x = -\sqrt{6}, -2$	A1	4	CAO, rejecting the positive
	Total		**15**	

Q	Solution	Marks	Total	Comments
6(a)	$\dfrac{\sec^2 x}{(\sec x+1)(\sec x-1)}=\dfrac{\sec^2 x}{\sec^2 x-1}$			
	$\sec^2 x = 1+\tan^2 x$ used	M1		M1 for correct use of $\sec^2 x = 1+\tan^2 x$ at least once or $\left(\text{cosec}^2 x = 1+\cot^2 x\right)$
	$=\dfrac{\sec^2 x}{\tan^2 x}$ or $\dfrac{1+\tan^2 x}{\tan^2 x}$			$\left(=\dfrac{1}{\cos^2 x\tan^2 x}\right)$
	$=\dfrac{1}{\sin^2 x}$ or $\cot^2 x+1$	A1		Shown, with no errors
	$=\text{cosec}^2 x$	A1	3	AG (No errors, omissions or poor notations seen)
(b)	$\text{cosec}^2 x=\text{cosec}\, x+3$ $\text{cosec}^2 x-\text{cosec}\, x-3=0$	B1		must have = 0
	$\text{cosec}\, x=\dfrac{1\pm\sqrt{13}}{2}$ or (2.3... **and** – 1.3...)	M1		correct solution of the quadratic, or by completing the square $\left(\text{cosec}\, x=\pm\sqrt{\dfrac{13}{4}}+\dfrac{1}{2}\right)$ PI by values for sin *x*
	$\sin x=\dfrac{2}{1\pm\sqrt{13}}$	B1F		B1F for $\text{cosec}\, x=\dfrac{1}{\sin x}$ seen or implied
	$=0.434$ **and** -0.768 $(\text{or } -0.767)$	A1		PI
	$x=26°, 154°, -50°, -130°$	B1 B1	6	B1 for any three values correct AWRT B1 for all four values correct AWRT and no extras in the interval $-180°<x<180°$
(c)	$2\theta-60°=x$	M1		where *x* is a written value from candidate's **(b)** in degrees PI by their answer
	$\theta=43°, 5°$	A1	2	CSO Ignore solutions outside interval $0°<\theta<90°$
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$y = 4x\cos 2x$			
	$\left(\frac{dy}{dx} =\right) 4\cos 2x - 4x(2)\sin 2x$	M1		anything reducible to $A\cos 2x + Bx\sin 2x$ where A and B are non-zero integers
		A1		OE, all correct
	gradient of the tangent $A\cos\frac{2\pi}{4} + B\times\frac{\pi}{4}\sin\frac{2\pi}{4}$	m1		substituting $\frac{\pi}{4}$ into candidate's derived function
	$= -2\pi$	A1		must have -2π using correct $\frac{dy}{dx}$
	an equation of the tangent is $y = -2\pi\left(x - \frac{\pi}{4}\right)$	A1	5	OE, dependent on previous A1
(b)				$\left(\int_0^{\frac{\pi}{4}} 4x\cos 2x\,dx\right)$
	$u = Ax \qquad \frac{dv}{dx} = \cos 2x$	M1		all 4 terms in this form seen or used
	$\frac{du}{dx} = A \qquad v = B\sin 2x$	A1		A = 4 and $B = \frac{1}{2}$ **or** A = 1 and B = 2, etc
	$= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \int_{(0)}^{\left(\frac{\pi}{4}\right)} 4\times\frac{1}{2}\sin 2x(dx)$	m1		correct substitution of candidate's terms into integration by parts formula condone missing limits
	$= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - [-\cos 2x]_{(0)}^{\left(\frac{\pi}{4}\right)}$	A1F		candidate's second integration completed correctly FT on one error including coefficient condone missing limits
	$= \frac{\pi}{2} - 1$	A1	5	OE, exact value
	Total		**10**	

Q	Solution	Marks	Total	Comments
8(a)	$\int e^{1-2x}dx = ke^{1-2x}$ or $e\left(ke^{-2x}\right)$	M1		where k is a rational number
	$\int_0^{\ln 2} e^{1-2x}dx = -\frac{1}{2}e^{1-2x}\Big\|_0^{\ln 2}$ or $e\left[-\frac{1}{2}e^{-2x}\right]_0^{\ln 2}$	A1		correct integration condone missing limits
	$= -\frac{1}{2}e^{1-2\ln 2} - -\frac{1}{2}e^{1-2(0)}$	A1		correct (no decimals)
	$= -\frac{1}{2}\left(\frac{1}{4}e\right) + \frac{1}{2}e$			eliminating ln
	$= \frac{3}{8}e$	A1	4	AG, be convinced
(b)	$u = \tan x$ $\frac{du}{dx} = \sec^2 x$	M1		PI below, condone $du = \sec^2 x\,dx$
	Replacing dx by $\frac{1}{\sec^2 x}(du)$ in integral	A1		or $\frac{1}{1+u^2}(du)$
	$\sec^2 x = 1 + u^2$	B1		PI below
	$x = 0 \Rightarrow u = 0$ $x = \frac{\pi}{4} \Rightarrow u = 1$	B1		this could be gained by changing u to tan x after the integration and using $x = 0$ and $x = \frac{\pi}{4}$
	$\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\,dx$ $= \int\left(1+u^2\right)\sqrt{u}\,(du)$ or $\int\left(1+u^2\right)^2\sqrt{u}\,\frac{(du)}{1+u^2}$	M1		all in terms of u including replacing dx all correct, condone omission of du
	$= \int\left(u^{\frac{5}{2}} + u^{\frac{1}{2}}\right)\,(du)$	A1		must be in this form
	$= \frac{2}{7}u^{\frac{7}{2}} + \frac{2}{3}u^{\frac{3}{2}}$	A1		accept correct unsimplified form
	$= \frac{20}{21}$	A1	8	CAO
	Total		**12**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2013

Mathematics **MPC3**

(Specification 6360)

Pure Core 3

Final

Mark Scheme

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Further copies of this Mark Scheme are available from: aqa.org.uk

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

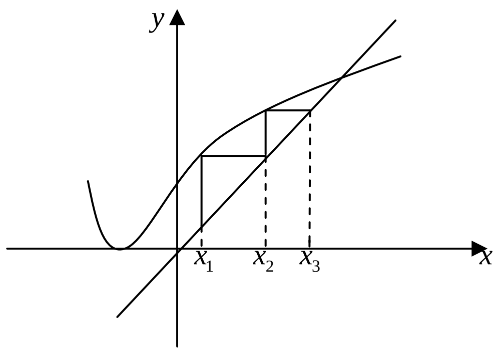
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

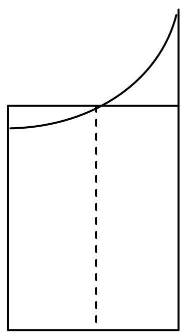
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

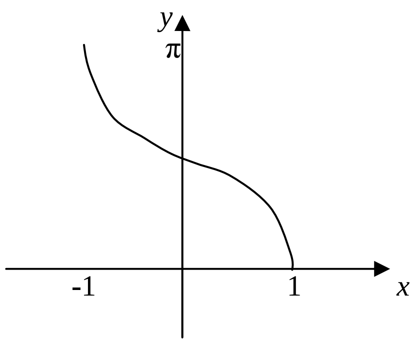
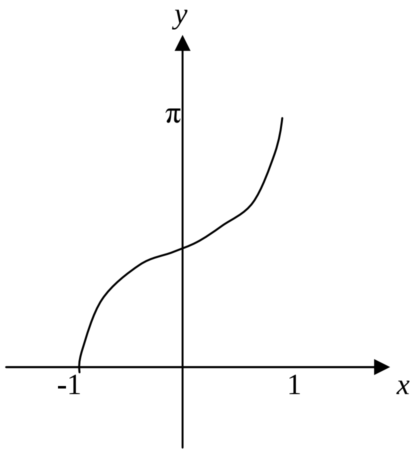
Otherwise we require evidence of a correct method for any marks to be awarded.

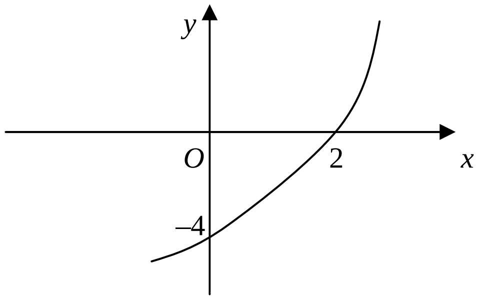
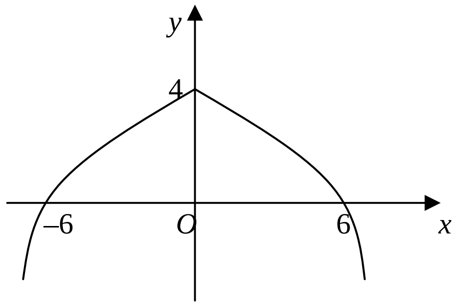
Q	Solution	Marks	Total	Comments
1(a)	$(2x-3=x)$			
	$x=3$	B1		
	$2x-3=-x$	M1		or $-(2x-3)=x$ or $-2x+3=x$
	$x=1$	A1	3	
(b)	$(\|2x-3\| \geq \|x\|)$			No ISW in part(b), ***mark their final line as their answer***.
	$x \leq 1$	B1		Or $1 \geq x$
	$x \geq 3$	B1	2	Or $3 \leq x$ Or "$x \leq 1$ *or* $x \geq 3$" for B1 B1
	Total		**5**	
2(a)	$(y = x^4 \tan 2x)$			
	$\left(\frac{dy}{dx}=\right)$ $4x^3 \tan 2x + x^4 2\sec^2 2x$	M1		$4x^3 \tan 2x + Ax^4 \sec^2 kx$ OE where A is a non-zero constant.
		A1		A1 for $k = 2$ may have $(\sec 2x)^2$ or $\frac{1}{\cos^2 2x}$
		A1	3	A1 all correct ISW if attempt to simplify is incorrect.
(b)	$\left(\frac{dy}{dx}=\right)\frac{\pm 2x(x-1) \pm 1(x^2)}{(x-1)^2}$	M1 A1		Use of the quotient rule $\frac{2x(x-1)-1(x^2)}{(x-1)^2}$
	$\left(=\frac{x^2-2x}{(x-1)^2}\right)$			**Simplification not required**
	$\left(\frac{dy}{dx}=\right)\frac{3}{4}$ or 0.75 OE	A1	3	Obtained from correct $\frac{dy}{dx}$
	Total		**6**	

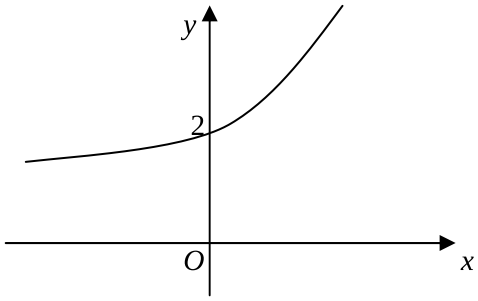
Q	Solution	Marks	Total	Comments
3(a)	$f(3) = -0.2(18)$ $f(4) = 0.01(83)$	M1		$f(x) = e^{-x} - 2 + \sqrt{x}$ Both values correct
	Change of sign $\Rightarrow$ $3 < \alpha < 4$	A1	2	Must have **both statement and interval** in words or symbols.
(b)	$\left(x_{n+1} = (2 - e^{-x_n})^2 \quad x_1 = 3.5\right)$ $\left(x_2 = 3.8801...\right)$			
	$x_2 = 3.880$	B1		Do **not** accept 3.88
	$\left(x_3 = 3.9178...\right)$			
	$x_3 = 3.918$	B1	2	Do **not** accept 3.917
(c)		M1		Staircase to curve from x_1 including at least two stairs between curve and line $y = x$.
		A1	2	x_2 and x_3 marked on the x-axis . Do **not** accept marking on the Curve or on the line.
	Total		**6**	

Q	Solution	Marks	Total	Comments
4	$\left(8\sec x - 2\sec^2 x = \tan^2 x - 2\right)$			
	$8\sec x - 2\sec^2 x = \sec^2 x - 1 - 2$	M1		Using $\tan^2 x = \sec^2 x - 1$ *and NOT replacing* $\sec^2 x$ with $1 + \tan^2 x$.
	$3\sec^2 x - 8\sec x - 3 (= 0)$	A1		
	$(3\sec x + 1)(\sec x - 3)(= 0)$ Or $\sec x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$	m1		Correct factors or correct use of quadratic equation formula or completing the square for 'their' equation. $\sec x - \dfrac{8}{6} = \pm\sqrt{\dfrac{64}{36} + 1}$
	$\sec x = 3, \; -\dfrac{1}{3}$ (or -0.33)	A1		Both correct.
	$\sec x = \dfrac{1}{\cos x}$ $\left(\cos x = \dfrac{1}{3} \text{ or } 0.33\right)$	B1		PI $\left(\sec x = -\dfrac{1}{3} \text{ is impossible}\right)$
	$x = 1.23, \; 5.05$	A1		One correct. Must have earned A1 for correct quadratic, but independent of the second A1.
		A1	7	Both correct and no extras in $0 < x < 2\pi$. CAO
	Total		**7**	

Q	Solution	Marks	Total	Comments
5(a)	x_i: $0.4(\frac{2}{5})$, $1.2(\frac{6}{5})$, $2(\frac{10}{5})$, $2.8(\frac{14}{5})$, $3.6(\frac{18}{5})$ y_i: 5.20231, 5.35985, 5.91608, 6.99657, 8.58231	B1		All 5 x-values correct, PI by 5 correct y-values.
		B1		At least 4 correct y-values rounded or truncated to at least 4 s.f. or in surd form $\sqrt{27+(0.4)^3}$, $\sqrt{27+(1.2)^3}$, etc. or $\sqrt{27.064}$, $\sqrt{28.728}$, etc. or sight of 32.057...
	$\int_0^4 \sqrt{27+x^3} \approx 0.8\sum_1^5 y_i$	M1		**Correct use of mid-ordinate rule** using 0.8 with candidate's 5 y-values. Dependent on first B1
	$(=0.8\times 32.057...)$			
	$=25.6$	A1	4	CAO (must be exactly this) **and no error seen**
(b)				Could be gained without answering part (a)
		B1		Diagram showing curve through the midpoint of the top of rectangle. May have one or more rectangles.
	"Smaller" OE	E1	2	Dependent on B1
	Total		**6**	

Q	Solution	Marks	Total	Comments
6(a)		B1		Correct sketch of $\cos^{-1} x$.
	$(-1,\ \pi)$ and $(1,\ 0)$	B1	2	Stated
(b)		B1		Correct sketch of $\pi - \cos^{-1} x$ Must touch negative x-axis.
	$(-1,\ 0)$ and $(1,\ \pi)$	B1	2	Stated
	Total		**4**	

Q	Solution	Marks	Total	Comments
7(a)		M1 A1	 2	Reflection in the x-axis. Intersection with the x-axis and y-axis marked 2 and –4. Accept (2, 0) and (0, –4) instead of marking on the axes.
(b)		M1 A1 A1	 3	Reflection of $0< x <6$ part in the y-axis giving two connected sections Correct curve beyond ± 6, correct curvature and correct cusp at x=0 (generous) ± 6 and 4 marked correctly Accept (6, 0), (0, 4) and (-6,0) instead of marking on the axes.
(c)	Reflection in the y-axis (followed by) 1) stretch 2) parallel to the x-axis 3) by factor 2 OR Vice versa	M1 A1 M1 A1	 4	 1 and either 2 or 3 1, 2 and 3
	Total		**9**	

Q	Solution	Marks	Total	Comments
8(a)(i)	$f(x) = \ln(2x-3)$			
	$2x - 3 = e^y$	M1		Either order: M1 for antilog
	$2y - 3 = e^x$	M1		M1 for replacing f(x) or y with x
	$\left(f^{-1}(x) =\right)\ \frac{1}{2}(e^x + 3)$ OE	A1	3	Correct expression in x
(ii)	$f^{-1}(x) > \frac{3}{2}$	B1	1	Do **not** condone $f^{-1}(x) \geq \frac{3}{2}$, $y > \frac{3}{2}$, $x > \frac{3}{2}$ range $> \frac{3}{2}$, $f^{-1} > \frac{3}{2}$
(iii)		M1		Correct shape crossing y-axis and above x-axis
		A1	2	2 marked on the y-axis
(b)(i)	$\left(gf(x) =\right)\ e^{2\ln(2x-3)} - 4$	M1		Correct composition
	$= e^{\ln(2x-3)^2} - 4$	m1		PI by correct expression
	$= (2x-3)^2 - 4$	A1	3	
(ii)	$\left(fg(x) =\right)\ \ln(2(e^{2x} - 4) - 3)$	M1		OE correct composition
	$\ln(2e^{2x} - 11) = \ln 5$			
	$2e^{2x} - 11 = 5$ OE	A1		Correct antilog of correct equation
	$e^{2x} = 8$			
	$2x = \ln 8$			
	$x = \frac{1}{2}\ln 8$	A1	3	OE **exact solution**, e.g. $\ln\sqrt{8}$ or $\frac{3}{2}\ln 2$ or $\ln 2^{\frac{3}{2}}$
	Total		**12**	

Q	Solution	Marks	Total	Comments
9				$V=\pi\int x^2\mathrm{d}y$ $16x^2-(y-8)^2=32$
	$x^2=\frac{1}{16}(y-8)^2+2$	B1		OE
	$V=(\pi)\int_{(0)}^{(16)}\left(\frac{1}{16}(y-8)^2+2\right)(\mathrm{d}y)$	M1		Accept 'their' x^2 in terms of y Condone missing limits and π **wherever bracketed**
	$V=(\pi)\left[\frac{1}{16}\times\frac{1}{3}(y-8)^3+2y\right]_{(0)}^{(16)}$	A1		OE, for correct integration of correct integrand
	$V=(\pi)\left[\frac{1}{16}\times\frac{1}{3}(16-8)^3+2(16)-\frac{1}{16}\times\frac{1}{3}(-8)^3\right]$	A1		OE, correct use of correct limits in correct expression, PI by correct answer.
	$V=\frac{160}{3}\pi$	A1	5	OE exact value, eg $\pi\,53\frac{1}{3}$ or $\pi\,53.\dot{3}$ or $\frac{2560}{48}\pi$
	Total		**5**	

Q	Solution	Marks	Total	Comments
10(a)(i)	$u = \ln x \quad \frac{dv}{dx} = 1$	M1		$\frac{d \ln x}{dx}$ & $\int dx$ attempted
	$\frac{du}{dx} = \frac{1}{x} \quad v = x$	A1		All correct
	$\left(\int \ln x \, dx =\right) \quad x \ln x - \int x \times \frac{1}{x} (dx)$	m1		Correct substitution of their terms into parts
	$= x \ln x - x + C$	A1	4	All correct (**constant needed**)
(ii)	$u = (\ln x)^2 \quad \frac{dv}{dx} = 1$	M1		$\frac{d(\ln x)^2}{dx}$ & $\int dx$ attempted
	$\frac{du}{dx} = (2 \ln x)\frac{1}{x} \quad v = x$	A1		All correct
	$\left(\int (\ln x)^2 dx =\right) \quad x(\ln x)^2 - \int x \times \frac{2}{x} \ln x \, (dx)$	m1		OE correct substitution of their terms into parts
	$= x(\ln x)^2 - 2(x \ln x - x) + C$ OE	A1	4	All correct (constant needed) including correct use of brackets. **Do not penalise missing constant if already penalised in part (i)** ISW
(b)	$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $\frac{1}{2} x^{-\frac{1}{2}}$ $(dx = 2u \, du)$	B1		$u = \sqrt{x}$
	$\int_{(1)}^{(4)} \frac{1}{x + \sqrt{x}} dx = \int_{(1)}^{(2)} \frac{1}{u^2 + u} 2u \, (du)$	M1		All in terms of *u* including attempt at replacing d*x* (not simply writing d*u*), condone missing limits and d*u*
		A1		Integrand correct unsimplified
	$= 2\int_{(1)}^{(2)} \frac{1}{u+1} (du)$	A1		
	$= 2 \ln(u+1) \Big\vert_{(1)}^{(2)}$	A1F		FT *their* $\int \frac{k}{u+1} (du)$
	$= 2 \ln(2+1) - 2 \ln(1+1)$ or $2 \ln(\sqrt{4} + 1) - 2 \ln(\sqrt{1} + 1)$	A1F		correct use of correct limits on $k \ln(u+1)$ or $k \ln\left(\sqrt{x} + 1\right)$
	$= 2 \ln \frac{3}{2}$ *or* $\ln \frac{9}{4}$ *or* $2 \ln 3 - 2 \ln 2$	A1	7	OE ISW
	Total		**15**	
	TOTAL		**75**	

A-LEVEL
MATHEMATICS
Pure Core 3 – MPC3
Mark scheme

6360
June 2014

Version/Stage: 1.0 Final

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Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	$y(0)=0$ $y\left(\frac{\pi}{4}\right)=0.6266(57068)$ $y\left(\frac{\pi}{2}\right)=1.253(314137)$ $y\left(\frac{3\pi}{4}\right)=1.085(401882)$ $y(\pi)=0$	B1 B1		All 5 x-values*, PI by 5 correct y-values All 5 y-values exact** or correct to at least 4SF (rounded or truncated)
	$\frac{1}{3}\times\frac{\pi}{4}\{0+0+4[0.6267+1.0854]+2[1.2533]\}$	M1		Correct use of Simpson's rule using $\frac{1}{3}\times\frac{\pi}{4}$ (or 0.26...) and 4 and 2 correctly with their 5 y-values from any x-values
	= 2.449	A1	4	CAO (must be exactly this value)
	Total		**4**	

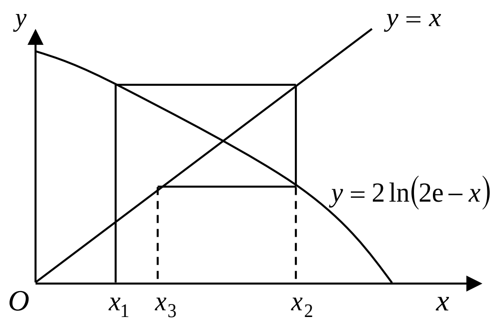
* Accept decimals 0.78(5398...), 1.5(7079...), 2.3(5619...), 3.1(4159...)

** $y\left(\frac{\pi}{4}\right)=\left(\frac{\pi}{4}\right)^{\frac{1}{2}}\sin\frac{\pi}{4}$, etc.

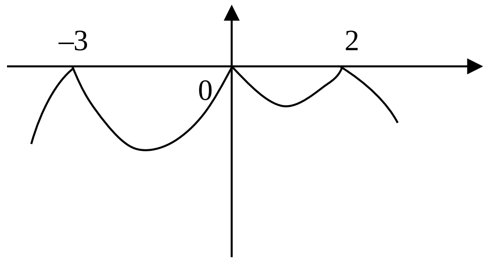
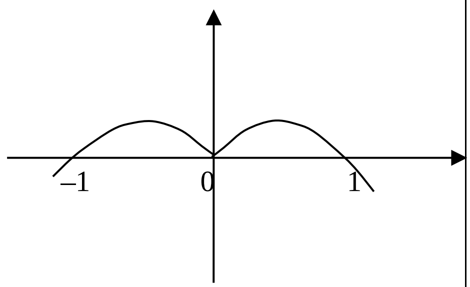
The minimum evidence for M1 is the 3 correct non-zero values of y in any form **and** sight of 2.4490(97...), but condone omission of the two zeros.

If a candidate's calculator setting is in degrees, they may earn the first B1 for 0, $\frac{\pi}{4}$, etc, and then B0, but M1 is available .

NMS: An answer of 2.449 without anything else gains 0/4.

Q	Solution	Mark	Total	Comment
2(a)	$\left(\frac{dy}{dx}=\right)\quad -2\times\frac{1}{(2e-x)}$ *or* $\frac{-2(2e-x)}{(2e-x)^2}$	M1		M1 for $\frac{k}{(2e-x)}$ or $k(2e-x)^{-1}, k\in$ □
		A1	2	OE, all correct
(b)	$\left(\frac{dy}{dx}=\right)\quad -2\times\frac{1}{(2e-e)}\quad\left(=-\frac{2}{e}\right)$	M1		Substituting e for x in **their** $\frac{dy}{dx}$ PI
	Gradient of the normal $=-\frac{2e-e}{-2}\quad\left(=\frac{e}{2}\right)$	m1		Must have also earned M1 in part (a)
	$y=2\ln(2e-e)\quad(=2)$	B1		
	$y-2=\frac{e}{2}(x-e)$ or $y=\frac{e}{2}x-\frac{e^2}{2}+2$	A1	4	OE, but must have simplified the gradient and replaced ln(2e–e) with 1
(c)(i)	$[f(x)=]\ 2\ln(2e-x)-x$ *or* $[g(x)=]\ x-2\ln(2e-x)$ $f(1)=1.98$ or $g(1)=-1.98$ $f(3)=-1.22$ or $g(3)=1.22$	M1		Must have both values correct rounded or truncated to 1 sf. Allow f(1) > 0 and f(3) < 0 only if f(x) is defined. **OR** evaluating **both** sides of $2\ln(2e-x)=x$: $2\ln(2e-1)=2.98$ $x=1$ $2\ln(2e-3)=1.78$ $x=3$ (M1) $2.98>1$ **and** $1.78<3\Rightarrow 1<\alpha<3$ (A1)
	Change of sign $\Rightarrow\ 1<\alpha<3$	A1	2	All working must be correct together with correct statement
(ii)	$x_2=2.980$ (2.97976...)	B1		Not 2.98
	$x_3=1.798$ (1.7977...)	B1	2	If B0, B0 scored but both values given correct to 3 sf or more than 3 dp, then SC1.
(iii)		M1		Vertical line from x_1 to curve(condone omission from x - axis to $y=x$) and then horizontal line from the curve to $y=x$ *
		A1	2	Second vertical and horizontal lines * and x_2 , x_3(or the values) must be labelled on x - axis **
	Total		**12**	
c(iii)	* On diagram, the solid lines may be dotted and the dotted lines need not be shown. ** Condone correct values (unrounded or 3 dp) marked on the x-axis instead of x_2 and x_3.			

Q	Solution	Mark	Total	Comment
3(a)(i)	$k\,x\left(x^2+1\right)^{\frac{3}{2}}$ or $k\,x\,u^{\frac{3}{2}}$	M1		Attempted use of the chain rule
	$\frac{5}{2}\times 2x\left(x^2+1\right)^{\frac{3}{2}}$	A1	2	OE, all correct
(ii)	$2e^{2x}$	B1		Differentiating e^{2x} correctly
	$\left(\frac{dy}{dx}=\right)\ 2e^{2x}\left(x^2+1\right)^{\frac{5}{2}}+e^{2x}(\text{their part (a)(i)})$	M1		OE
	$\left(\frac{dy}{dx}=\right)\ 2e^{2x}\left(x^2+1\right)^{\frac{5}{2}}+e^{2x}\frac{5}{2}\times 2x\left(x^2+1\right)^{\frac{3}{2}}$			
	$(\text{When } x=0\ )\quad \frac{dy}{dx}=2$	A1	3	Substituting $x = 0$ and CSO
(b)	$\frac{4\left(x^2+1\right)-2x\left(4x-3\right)}{\left(x^2+1\right)^2}$	M1		M1 for $\frac{\pm 4\left(x^2+1\right)\pm 2x\left(4x-3\right)}{\left(x^2+1\right)^2}$
		A1		All correct
	$-4x^2+6x+4\ (=0)$ *or* $2x^2-3x-2\ (=0)$	m1		Forming a quadratic equation with all terms on one side $ax^2+bx+c\,(=0)$ $b\neq 0,\ c\neq 0$
	$(2x+1)(x-2)\ (=0)$	A1		OE correct factors or using the formula as far as $x=\frac{3\pm\sqrt{25}}{4}$ or completing the square as far as $x-\frac{3}{4}=\pm\sqrt{\frac{25}{16}}$
	$x=2\ ,\quad x=-\frac{1}{2}$	A1	5	CAO, simplified answers
	Total		**10**	

Q	Solution	Mark	Total	Comment
4(a)		M1		Reflection in the x-axis for the positive f(x) and the remainder as given in the sketch.
		A1		Correct $-3 < x < 2$ with minimum at $x < 0$ lower than minimum at $x > 0$ and correct cusps at $x = -3, 0, 2$.
		A1	3	Correct branches for $x > 2$ and $x < -3$, including the curvature of both branches and 2 and –3 marked *
(b)		M1		Symmetrical about the y-axis using only the original curve for $x>0$
		A1	2	–1 and 1 labelled on the x-axis and correct cusp at $x=0$
(c)(i)*	Stretch (I) s.f. $\frac{1}{2}$ (II) // x-axis (III)	M1		(I) and either (II) or (III)
		A1		(I) and (II) and (III)
	(followed by) Translation	E1		Not 'shift', 'move', etc.
	$\begin{bmatrix} -1 \\ 0 \end{bmatrix}$	B1	4	Or in words Not (–1, 0)
(ii)	(1, –3) Or $x = 1$, $y = -3$	B1 B1	2	B1 for each coordinate
	Total		**11**	

(a) * The two A1 marks are independent. Condone straight lines for the branches for $x>2$ and $x<-3$ but not curves which are concave upward.

***(c)(i)** Alternative:

Translation E1

$\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ B1

(followed by)

Stretch (I)

s.f. $\frac{1}{2}$ (II)

// $x-$axis (III)

M1 : (I) and either (II) or (III) A1: (I), (II) and (III)

Q	Solution	Mark	Total	Comment
5(a)		M1		$f(x) > -4$, *or* $****** \geq -4$,
	$f(x) \geq -4$	A1	2	
(b)	$y = (x-3)^2 - 4$			
	$x - 3 = (\pm)\sqrt{y+4}$	M1		
	$x = 3 \pm \sqrt{y+4}$	A1		condone $x = 3 + \sqrt{y+4}$
	$y = 3 \pm \sqrt{x+4}$	B1		interchanging *x* and *y* at any stage
	$\left(f^{-1}(x) =\right) \quad 3 + \sqrt{x+4}$	A1	4	negative clearly rejected. must have $\pm$ earlier.
(c)(i)	$\left(gf(x) =\right) \quad \lvert x^2 - 6x + 5 - 6 \rvert$ or $\lvert x^2 - 6x - 1 \rvert$	B1	1	
(ii)	$'their\ x^2 - 6x + 5 - 6' = 6$	M1		and attempt to solve 3 term quadratic
	$'their\ x^2 - 6x + 5 - 6' = -6$	M1		and attempt to solve 3 term quadratic
	$x = 7$ $x = -1$ $x = 5$ $x = 1$	A1		all four solutions seen and correct
	$x = 5, \quad x = 7$	E1	4	values 1 and –1 clearly rejected
	Total		**11**	

(a) $f(x) > -4$, $f \geq -4$, ≥ -4 , $x \geq -4$, range ≥ -4 , $y \geq -4$ score M1 only
$y > -4$, etc scores M0 (two errors)

(b) Alternative

$y = x^2 - 6x + 5$

$x^2 - 6x + (5 - y) = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(5-y)}}{2}$ correctly solving M1

$x = \dfrac{6 \pm \sqrt{16 + 4y}}{2}$ A1

B1 for swapping *x* and *y* and A1 for $\dfrac{6 + \sqrt{16 + 4x}}{2}$ having rejected minus sign

Q	Solution	Mark	Total	Comment
6(a)	$\int x^2 \sin 2x \, dx$			
	$u = x^2 \quad \frac{du}{dx} = 2x$; $\frac{dv}{dx} = \sin 2x \quad v = -\frac{1}{2}\cos 2x$	M1		$\frac{du}{dx} = kx, k = 1 \text{ or } 2$ and $v = p\cos 2x$ $p = \pm 1, \pm 2, \pm 0.5$
		A1		All correct
	$\left(\int x^2 \sin 2x \, dx =\right)$ $-\frac{1}{2}x^2 \cos 2x + \int x \cos 2x (dx)$	A1F		Correct substitution of **their** terms into parts formula
	$u = x \quad \frac{du}{dx} = 1$; $\frac{dv}{dx} = \cos 2x \quad v = \frac{1}{2}\sin 2x$	m1		Correct follow through unsimplified from their first integral above
	$\left(\int x^2 \sin 2x \, dx = \right)$ $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x - \frac{1}{2}\int \sin 2x (dx)$	A1		Correct
	$= -\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{2} \times \frac{1}{2}\cos 2x + C$	A1		OE, **must** have constant of integration
			6	
(b)	$(V =) \quad \pi \int_0^{\frac{\pi}{2}} x^2 \sin 2x \, dx$	B1		Fully correct including dx and limits
	$= (\pi)\left[-\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \cos \pi + \frac{1}{2}\left(\frac{\pi}{2}\right)\sin \pi + \frac{1}{4}\cos \pi - \frac{1}{4}\right]$	M1		Attempt at $F\left(\frac{\pi}{2}\right) - F(0)$ FT their expression from part (a)
	$= \pi\left(\frac{\pi^2}{8} - \frac{1}{2}\right)$	A1		OE in exact form with $\cos \pi$ and $\sin \pi$ evaluated
			3	
	Total		**9**	

Q	Solution	Mark	Total	Comment
7	$\frac{du}{dx} = -3x^2$ or $du = -3x^2\ dx$ and substituting for dx and x in terms of u	M1		Condone $\frac{du}{dx} = 3x^2$ or $du = 3x^2 dx$ for M1
	$\int \frac{-(3-u)}{3u}\ du$	A1		OE correct unsimplified integral in terms of u only with du seen on this line or later
	$= \int \left(\frac{1}{3} - \frac{1}{u}\right)(du)$	A1		PI by the next line
	$= \left[\frac{u}{3} - \ln u\right]_{(3)}^{(2)}$	A1F		FT on their $\int \left(a + \frac{b}{u}\right)\ du$
	$= \left[\frac{2}{3} - \ln 2 - \left(\frac{3}{3} - \ln 3\right)\right]$	m1		Correct use of correct limits in u for expression of form $au + b\ln u$ or in terms of x
	$-\ln 2 + \ln 3 - \frac{1}{3}$ *or* $\ln\frac{3}{2} - \frac{1}{3}$	A1	6	OE exact value
	Total		**6**	

Q	Solution	Mark	Total	Comment
8(a)	$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=\frac{(1-\sin x)^2+\cos^2 x}{\cos x(1-\sin x)}$	M1		Combining fractions correctly
	$=\frac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$			
	$=\frac{1-2\sin x+1}{\cos x(1-\sin x)}$	m1		Using $\sin^2 x+\cos^2 x=1$
	$=\frac{2-2\sin x}{\cos x(1-\sin x)}$ *or* $\frac{2(1-\sin x)}{\cos x(1-\sin x)}$	A1		Must have factorised denominator
	$=\frac{2}{\cos x}$			
	$=2\sec x$	A1	4	AG, both expressions seen
(b)	$\tan^2 x-2=2\sec x$			
	$\sec^2 x-1-2=2\sec x$			Using $\tan^2 x=\sec^2 x-1$, OE
	$\sec^2 x-2\sec x-3\ (=0)$	B1		Or $3\cos^2 x+2\cos x-1\ (=0)$
	$(\sec x-3)(\sec x+1)\quad(=0)$	M1		Correctly factorising their expression or substituting into formula
	$\sec x=3$ *or* -1	A1		Or $\cos x=\frac{1}{3}$ *or* -1
	$\sec x=3 \Rightarrow x=71°,\ 289°$	B1 B1		no extras inside the interval $0\le x<360°$, −1 EE
	$\sec x=-1 \Rightarrow x=180°$	B1	6	
(c)	$2\theta-30°=70.5°,\ 180°,\ 289.5°$	M1		For RHS accept any *x*-value from part (b) PI
	$\theta=50°,\ 105°,\ 160°$	A1	2	Allow $51°,\ 105°,\ 160°$
	Total		**12**	
	TOTAL		**75**	

(b) $x=70°$ and $290°$ scores B0 B0
AWRT $x=71°$ and $289°$ both not given to the nearest degree earns **SC1**.

(c) Condone correct answers not given to the nearest degree **if already penalised in part (b)**,
AWRT $\theta=50°$ *or* $51°$, $105°$, $160°$

A-LEVEL

Mathematics

Pure Core 3 – MPC3
Mark scheme

6360
June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

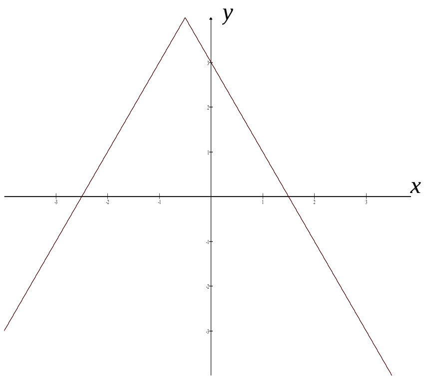
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
1a	| x | y | | 2 | $1 \times \ln 4 = 1.38629$ | | 3 | $e^{-1} \times \ln 7 = 0.71586$ | | 4 | $e^{-2} \times \ln 10 = 0.31162$ | | 5 | $e^{-3} \times \ln 13 = 0.12770$ |	B1		All 4 correct x values (and no extras used) PI by 4 correct y values
		M1		At least 3 correct y in exact form or decimal values, rounded or truncated to 3dp or better (in table or formula) (PI by correct answer)
	$\int = (1 \times)\sum y$	m1		Correct substitution into formula, with h=1 of 4, and only 4, correct y values (as above) either listed (with + signs) or totalled.
	$= 2.541$	A1	4	CAO, must be this exactly and no error seen
b	$(\frac{dy}{dx}=) -e^{2-x}\ln(3x-2)+e^{2-x}\frac{3}{3x-2}$	M1		$Ae^{2-x}\ln(3x-2)+e^{2-x}\frac{B}{3x-2}$
		A1		$A=-1$
		A1		$B=3$
	(When $x=2$) $(\frac{dy}{dx}=)\frac{3}{4}-\ln 4$ or $\frac{3}{4}+\ln\frac{1}{4}$	A1	4	ISW
	Total		8	

(a) NMS: An answer of 2.541 without anything else earns 0/4
The '1 x' may not be seen but implied

(b) NMS: An answer of -0.636 without anything else earns 0/4

Q2	Solution	Mark	Total	Comment
a		M1 A1		Correct shape, inverted V, roughly symmetrical, with vertex in the 2nd quadrant In all 4 quadrants
	(1.5, 0) **and** (-2.5, 0) (0, 3)	B1 B1	4	Shown on sketch or coordinates stated Shown on sketch or coordinates stated (diagram takes precedence)
b	$(x=)1$ $x=4+(2x+1)$ $(x=)-5$	B1 M1 A1	3	OE
c	$-5<x<1$	B2	2	Or for $x>-5$ **AND** $x<1$
d	Reflection in $y=k$ x-axis (or line $y=0$) (followed by) Translation $\begin{bmatrix}0\\p\end{bmatrix}$ $p=4$	M1 A1 M1 A1	4	Translation $\begin{bmatrix}0\\p\end{bmatrix}$ (M1) $p=4$ (A1) (followed by) (PI) Reflection in $y=k$ (M1) $k=4$ (A1) oe
	Total		**13**	

(a) For **M1** must be attempt at straight lines. Condone correct values on axes for **B1, B1**

(b) NMS: $x=-5$ scores **SC1**
If squaring: $x^2-8x+16=4x^2+4x+1$ therefore $3x^2+12x-15=0$ scores **M1**, then **A1, B1** as above

(c) $x>-5, x<1$ scores **SC1** $x>-5$ or $x<1$ scores **SC1**
SC1 for $-5\le x\le 1$ or $-5\le x\le 1$ or $-5\le x\le 1$

(d) There are other correct possible transformations, but for full marks the order of the two transformations must produce the correct answer.

Q3	Solution	Mark	Total	Comment
ai	$f(x) = 6\ln x - 8x + x^2 + 3$ f(5) = - 2.3 f(6) = 1.75 Change of sign(or different signs) $\Rightarrow 5 < \alpha < 6$	M1 A1	 2	(or reverse) Both values correct to 1sf (rounded or truncated) Must have both statement and interval in words or symbols AND f(x) defined **OR** comparing 2 sides: $6\ln 5 = 9.7 \quad 8\times 5 - 5^2 - 3 = 12$ $6\ln 6 = 11 \quad 8\times 6 - 6^2 - 3 = 9$ (M1) at 5, LHS < RHS; at 6 LHS > RHS $\Rightarrow 5 < \alpha < 6$ (A1)
ii	$x = 4 + \sqrt{13 - 6\ln x}$ $x - 4 = \sqrt{13 - 6\ln x}$ $(x-4)^2 = 13 - 6\ln x$ $x^2 - 8x + 16 = 13 - 6\ln x$ $6\ln x + x^2 - 8x + 3 = 0$	M1 A1 A1	 3	Correctly eliminate square root Must see squared term correctly expanded AG, CSO
iii	$x_2 = 5.828$ $x_3 = 5.557$	B1 B1	2	
bi	$\frac{dy}{dx} = \frac{6}{x} + 2x - 8$ $(\frac{dy}{dx} = 0)\ \ 6 + 2x^2 - 8x = 0$ $x = 1, \quad x = 3$ $(x = 1), \quad y = -4$ $(x = 3), \quad y = 6\ln 3 - 12$ **or** $\ln 729 - 12$	B1 M1 A1 A1 A1	 5	Condone $\frac{6x^5}{x^6}$ Equate to zero (PI) and eliminate their fraction correctly. Oe for other exact correct values If **M0** then SC1 for (1, -4) and/or $(3,\ 6\ln 3 - 12)$
ii	$x = 5, \quad y = -8$ $x = 7, \quad y = 12\ln 3 - 24$	M1 A1	 2	their x + 4 and $2\times$their y on either of their 'pairs' All correct : oe exact
	Total		**14**	

(a)(ii) Condone all terms in any order on one side but must have =0
(a)(iii) No credit for any answers not to this accuracy

Q4	Solution	Mark	Total	Comment
a		M1		$f(x) \le 5$, ** < 5
	$f(x) < 5$	A1	2	
bi	$x = 5 - e^{3y}$	M1		Swap x and y at any stage.
	$e^{3y} = 5 - x$			
	$3y = \ln(5 - x)$	M1		Correctly converting to ln.
	$(f^{-1}(x) =) \frac{1}{3}\ln(5 - x)$	A1	3	ACF
ii	$(x =)\ \ 4$	B1	1	
c	$[gg(x) =]\ \frac{1}{2\left(\frac{1}{2x-3}\right) - 3}$	M1		
	$= \frac{1}{\frac{2 - 6x + 9}{2x - 3}}$	A1		or $\frac{2x-3}{2-3(2x-3)}$
	$= \frac{2x - 3}{11 - 6x}$	A1	3	
	Total		9	

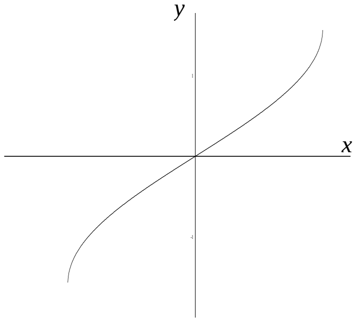
(b)(i) Must be convinced that final answer is not $\ln\frac{5-x}{3}$ or $\ln(5-x)/3$

Q5	Solution	Mark	Total	Comment
a	$(\frac{dy}{dx}=)\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$	M1		$\frac{\pm\cos^2 x\pm\sin^2 x}{\cos^2 x}$
	$=\frac{1}{\cos^2 x}$ or $1+\tan^2 x$			Must see this line
	$=\sec^2 x$	A1	2	AG; no errors seen and all notation correct
b	$\int x\sec^2 x\mathrm{d}x$			
	$u=x$ $\quad \frac{dv}{(dx)}=\sec^2 x$			
	$\frac{du}{(dx)}=1$ $\quad v=\tan x$	M1		All 4 terms in this form with $\frac{du}{dx}$ correct and $\int\frac{dv}{dx}$ attempted
	$v=\tan x$	B1		
	$x\tan x-\int\tan x(\mathrm{d}x)$	A1		
	$=x\tan x-\ln\sec x+c$	A1	4	OE (e.g. $x\tan x+\ln\cos x$); must have constant of integration
c	$(V=)\pi\int_0^1 25x\sec^2 x\mathrm{d}x$	B1		Must include π, limits and dx
	$=(25\pi)[(1\tan 1-\ln\sec 1)-0]$	M1		Must have $(k)\int x\sec^2 x$ then correct substitution of 0 and 1 into $ax\tan x+b\ln(\sec\ or\ \cos)x$ Condone missing 0.
	$=74$	A1	3	Condone AWRT 74
	Total		9	

(a) Use of product rule scores **M0**

(c) $\left[(5\sqrt{x})\sec x\right]^2$ must be correctly expanded for B1 to be available.

If the integration has been re-started, then **M1** must be for substitution into $ax\tan x+b\ln\sec x$

Q6	Solution	Mark	Total	Comment
a		B1		Correct shape passing through origin
	$\left(\frac{1}{3}, \frac{\pi}{2}\right)$	B1		Must be stated
	$\left(-\frac{1}{3}, -\frac{\pi}{2}\right)$	B1	3	Must be stated
b	$\frac{dx}{dy} = \frac{1}{3}\cos y$	M1		
	$\frac{dy}{dx} = \frac{3}{\cos y}$ or $3\sec y$	A1	2	Both $\frac{dx}{dy}$ and $\frac{dy}{dx}$ seen and used correctly
	Total		**5**	

(a) Coordinates must be stated NOT just indicated on axes, but BOTH correct end points clearly labelled on axes scores **SC1.**

Q7	Solution	Mark	Total	Comment
	$\frac{du}{dx}=-2x$ or $du=-2x\,dx$	M1		Condone $\frac{du}{dx}=2x$ or $du=2x\,dx$
	$\int\frac{6-u}{u^{0.5}}\times\frac{du}{-2}$	A1		OE correct unsimplified integral in terms of u only, with du seen on this line or later
	$=-\frac{1}{2}\int(6u^{-0.5}-u^{0.5})du$	m1		Terms in the form $\int(au^{-0.5}+bu^{0.5})du$
	$=-\frac{1}{2}(6\frac{u^{0.5}}{0.5}-\frac{2u^{1.5}}{3})$	A1F		Ft must be in the form $cu^{0.5}+du^{1.5}$ Oe (eg allow $c\sqrt{u}$)
	($=-6u^{0.5}+\frac{1}{3}u^{1.5}$)			
	(Limits $[x]_1^2=[u]_5^2$)			
	$\int_5^2=\left[-6u^{0.5}+\frac{1}{3}u^{1.5}\right]_5^2$ $=(-6\times2^{0.5}+\frac{1}{3}\times2^{1.5})-(-6\times5^{0.5}+\frac{1}{3}\times5^{1.5})$	m1		Correct substitution into expression of the form $eu^{0.5}+fu^{1.5}$ and F(2) – F(5), or if using x, F(2) – F(1)
	$=\frac{13}{3}\sqrt{5}-\frac{16}{3}\sqrt{2}$	A1A1	7	oe any correct exact form
	Total		7	

For first A1 allow: $\int\frac{(6-u)^{\frac{3}{2}}}{\sqrt{u}(6-u)^{\frac{1}{2}}}\times\frac{du}{-2}$

For second m1 the substitution must be in the correct order

Q8	Solution	Mark	Total	Comment
a	$LHS = 4(1+\cot^2\theta) - \cot^2\theta$	**M1**		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$4(1+\cot^2\theta) - \cot^2\theta = k$ Or $4\operatorname{cosec}^2\theta - (\operatorname{cosec}^2 - 1) = k$	**A1**		All correct, including = k
	$\cot^2\theta = \dfrac{k-4}{3}$	**m1**		Correctly isolating trig function – must be tan or cot or cos or sec, from their **CORRECT** equation
	$\tan^2\theta = \dfrac{3}{k-4}$	**m1**		Correct inversion (at some stage) from their equation
	$[\sec^2\theta = \dfrac{3}{k-4} + 1]$			Must see at least one line of working, be convinced
	$\sec^2\theta = \dfrac{k-1}{k-4}$	**A1**	**5**	AG: no errors seen
b	$\sec^2\theta = 4$ or $\tan^2\theta = 3$ or $\cot^2\theta = \dfrac{1}{3}$ or $\operatorname{cosec}^2\theta = \dfrac{4}{3}$	**B1**		PI by expression for eg $\sec x = 2$
	$\sec\theta = \pm 2$	**M1**		or $\cos\theta = \pm 0.5$ or $\tan\theta = \pm\sqrt{3}$ or $\sin\theta = \pm\dfrac{\sqrt{3}}{2}$
	$(\theta =)$ 60, 120, 240, 300, 420	**A1**		Sight of any four of these answers
	$x = 22.5°, 82.5°, 112.5°, 172.5°$	**B1** **B1**	**5**	3 correct All correct and no extras in interval (ignore answers outside interval)
	Total		**10**	

(a) The two **m1** marks can be earned in either order.
There are many different approaches

(b) If working in radians then max mark is **B1, M1**

(a)	Different approaches: **I** $LHS = 4(1+\cot^2\theta) - \cot^2\theta$	**M1**		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$4(1+\cot^2\theta) - \cot^2\theta = k$	**A1**		All correct, including $= k$
	$k-1 = 3+3\cot^2\theta$ $k-4 = 3\cot^2\theta$	**m1**		Both correct equations from their equation in *k*.
	$\dfrac{k-1}{k-4} = \dfrac{3+3\cot^2\theta}{3\cot^2\theta}$	**m1**		Correct equation from their 2 previous equations
	$\sec^2\theta = \dfrac{k-1}{k-4}$	**A1**		AG: no errors seen
	II $LHS = \dfrac{4}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$ $= \dfrac{4-\cos^2\theta}{1-\cos^2\theta}$	**M1**		Use of a correct trig identity (or identities if using sin/cos) to get an expression/equation in a single trig function
	$\dfrac{4-\cos^2\theta}{1-\cos^2\theta} = k$	**A1**		All correct, including = k
	$\dfrac{4\sec^2\theta - 1}{\sec^2\theta - 1} = k$	**m1**		Correct 'inversion' (at some stage) from their equation
	$4\sec^2\theta - 1 = k\sec^2\theta - k$			Must see at least one line of working, be convinced for final A1
	$k-1 = \sec^2\theta(k-4)$	**m1**		Correct equation in the form $a\sec^2\theta = b$ *or* $a\cos^2\theta = b$ from their **CORRECT** equation
	$\sec^2\theta = \dfrac{k-1}{k-4}$	**A1**		AG: no errors seen